A NUMERICAL MODEL OF SEDIMENT DEPOSITION ON SALTMARSHES

by

Steven James Woolnough

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University of Reading,

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A one-dimensional model of sediment transport and deposition over a saltmarsh is developed by simplifying a three-dimensional mass balance equation for the sediment and integrating the resultant equation over the depth of the flow . The

Contents

Notation

A	amplitude of tide [m]
b	height of saltmarsh above mean sea-level [m]
b'	non-dimensionalised b
C	fractional volume of sediment in fluid
C_0	fractional volume of sediment in tidal water
	diameter of sediment particle [m]
E_s	absolute error in the depth of sediment deposited [m]
H	height of water above saltmarsh [m]
H'	non-dimensionalised H
j	label of space grid points
j_c	node before x_c^P
j_E	node before x_E
k	mode number of fourier decomposition of y
L	length of saltmarsh [m]
M	number of space steps
N	half the number of time steps
n	exponent or time level
P	number of time steps in characteristic solving routine
p	time level in characteristic solving routine
R	rate of deposition on saltmarsh [ms^{-1}]
r_M	gradient of line of best fit for graph of error against M
r_N	gradient of line of best fit for graph of error against N
S	depth of sediment deposited on the saltmarsh [m]
S_j^n	approximation to depth of sediment deposited on the marsh at (x_j, t_n) [m]
S'	non-dimensionalised S
T	tidal period [s]
t	time [s]
t_E	time at which no sediment is suspended above a point on the marsh [s]
t'	non-dimensionalised t
t_n	time at nth time level [s]
t_p	time at pth time level of characteristic solving routine [s]
t_0	phase of tide at which water covers the saltmarsh [s]
t'_0	non-dimensionalised t_0
u	horizontal fluid velocity [ms^{-1}]
u'	non-dimensionalised u

u_i	fluid velocity components $[ms^{-1}]$
$[uy]_j^n$	approximation to $uy(x_j, t_n)$ $[m^2 s^{-1}]$
v_p	particle settling velocity $[ms^{-1}]$
v_p'	non-dimensionalised v_p
v_0	settling velocity of single smooth sphere $[ms^{-1}]$
w	vertical fluid velocity $[ms^{-1}]$
x	horizontal coordinate $[m]$
x'	non-dimensionalised x
x_c	position of characteristic through x_E $[m]$
x_c^p	approximation to $x_c(t_p)$ $[m]$
$x_c^{p,(k)}$	kth iteration of x_c^p $[m]$
x_j	position of jth node $[m]$
x_E	position at which no sediment is suspended above the marsh $[m]$
x_i	components of position vector $[m]$
y	height of column of sediment in water
y'	non-dimensionalised y
y_c	value of y on characteristic x_c $[m]$
$y_{c'}$	approximation to $y(x_c^P, t_n)$ $[m]$
y_j^n	approximation to $y(x_j, t_n)$ $[m]$
z	vertical coordinate $[m]$
α	label of characteristics
ϵ_i	sediment mixing coefficients $[m^2 s^{-1}]$
η	dynamic viscosity of fluid $[Nsm^{-2}]$
θ	parameter in box scheme
λ	the Courant number
ξ	amplitude of fourier mode
ρ	density of fluid $[kgm^{-3}]$
σ	density of sediment $[kgm^{-3}]$
τ	truncation error
ϕ	parameter in box scheme
ω	angular frequency of tide $[s^{-1}]$
Δt	time step [s]
Δt_c	time step in characteristic solving routine $[s]$
Δx	space step $[m]$
$\Delta x'$	space step for first step in box scheme for ebb tide $[m]$

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Introduction

1.1 What is a Saltmarsh?

Coastal saltmarshes are relatively flat areas of land which are regularly flooded by the sea; they occur high in the intertidal zone, mainly in temperate and high latitudes on low energy coasts [Allen and Pye 1992]. Their occurrence is controlled by the coastal geography since deposited sediment can only accumulate where the wave action is small. Hence saltmarshes tend to be found only in sheltered areas like bays and estuaries or on the lee side of spits and barrier islands. An exception to this is where a major river deposits fine sediment which forms a large and shallow region close to the shore which reduces the intensity of the incoming waves, for example, the Mississippi Delta.

There are several processes which affect the development of saltmarshes. Firstly, they need a source of sediment: this is usually from the suspended material in the tidal water which periodically floods the marshes. The sediment will only be deposited when the fluid velocities are small. Hence a saltmarsh will only develop near the top of the intertidal region which is only covered at slack water. The presence of vegetation on the marsh will affect the growth of the marsh in two farming arable crops.

Until recently however they have been of little interest to coastal engineers, but in the past few years there has been much attention paid to global warming and the resultant threat of rising sea-levels and a stormier climate on the lowlying coastal regions of Britain. The danger of loss of land to the sea has brought about a need to improve the coastal defences around much of the British Isles.

Traditional coastal defence tactics such as the construction of sea walls can be expensive and cheaper methods of protecting the land are being considered. One such tactic is the use of the natural features of the coastline as a defence mechanism. The most important feature of the saltmarsh in this respect is the way in which they dissipate much of the incoming wave energy so that little remains at the landward end [Brampton 1992]. This enables the land beyond to be protected from the sea by a much smaller and therefore cheaper wall. To use saltmarshes as an effective aid to coastal defence, without damaging their ecological value, requires an understanding of the ways in which the marsh develops and how human interference may affect this. This project uses a simplified one-dimensional model of sediment transport over the marsh to simulate numerically the deposition of sediment on the marsh by the tide.

2.1.1 Settling Velocities

The settling velocity of a single smooth spherical particle in a stagnant unbounded fluid for small particle Reynolds numbers, $Re = \rho \quad v_0/\eta$, is given by Stokes' Law

$$v_0 = \frac{1}{18}\frac{(\sigma - \rho)g \quad ^2}{\eta} \tag{2.2}$$

where v_0 is the settling velocity of the single particle, σ is the particle density, ρ is the fluid density, g is the acceleration due to gravity, is the particle diameter and η the dynamic viscosity of the fluid.

Richardson and Zaki(1954) find by experiment that the relationship between settling velocity for an array of particles and a single particle is given by

$$v_p = v_0(1 - C)^n \tag{2.3}$$

where n is a positive exponent dependent on the particle Reynolds number. Maude and Whitmore(1958) find theoretically that $2.33 \leq n \leq 4.65$ and generate a curve of the relationship between n and the Reynolds number which agrees well with the experimental results of Richardson and Zaki, as shown by Allen(1985).

Similarly Hallemeier(1981) suggests a scheme, based on experimental results, for modifying the settling velocity for non-spherical particles. This scheme has little effect on settling velocities in low Reynolds number cases.

2.1.2 Fluid Velocity Components

To solve equation 2.1 a knowledge of the fluid velocities is required, these can be obtained from analytical or numerical solutions of the equations of motion of the fluid or approximations to them. These fluid velocities will be specific to the problem being considered.

The sediment mixing coefficients are related to the diffusivities for the momentum of the fluid. There are many ways of modelling turbulence and calculating diffusion coefficients, based on results from both theory, eg. Prandtl's mixing length theory [Graf 1971] or experiments, eg. Rajaratnam and Ahmadi(1981). Once again these results will be dependent on the problem being considered.

In order to solve equation 2.1 in a finite region boundary conditions must be specified. These are based on the physical boundary conditions that no sediment may be transferred across the water surface. A known concentration profile may be specified at a boundary of the region. At a solid boundary, eg. the bed of the region, the rate of transfer is defined by the probabilities of a particle reaching the boundary being deposited and of a particle on the boundary being eroded. At a vertical boundary it can usually be assumed that there is no transfer of sediment. At the bed of the region the probabilities can be estimated in many ways James(1987) ignores the possibility of erosion and defines the probability of deposition by the complement of the erosion probability defined by instein(1950).

The saltmarsh is assumed to be horizontal and alongside a body of tidal water, i.e.

transverse direction need to be considered. Figure 2.1 shows the two-dimensional approximation to the region of investigation.

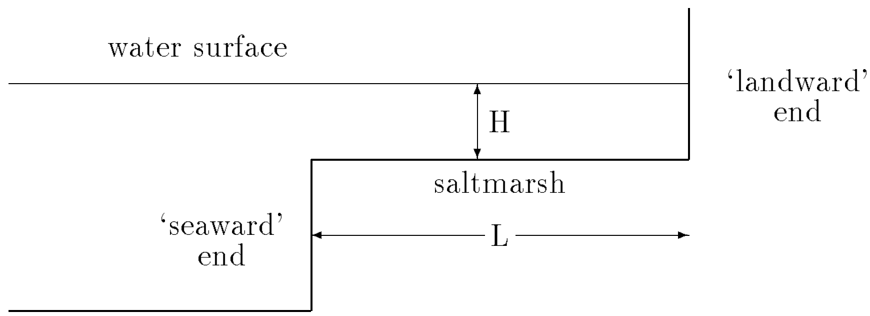


Figure 2.1: The region of investigation

The marsh is bounded at one end, hereafter known as the 'seaward end', by the body of water, which is the source of the sediment and at the 'landward' end, a distance L away, by a vertical barrier. It is assumed that the water surface remains horizontal across the marsh, this implies that the tide, when it reaches the level of the marsh, instantaneously covers it. The depth of the water, H, over the marsh is therefore uniform and a function of time only.

The simplifications made here allow the removal of one of the dimensions from equation 2.1. The mass balance equation for suspended sediment in two dimensions is

$$\frac{\partial C}{\partial t} + \frac{\partial}{\partial x}(uC) + \frac{\partial}{\partial z}(wc) = \frac{\partial}{\partial x}\left(\epsilon_x \frac{\partial C}{\partial x}\right) + \frac{\partial}{\partial z}\left(\epsilon_z \frac{\partial C}{\partial z}\right) + \frac{\partial}{\partial z}(v_p C) \qquad (2.4)$$

where x is the transverse direction and u and w are the velocities in the x and z directions, with the origin on the marsh at the seaward end.

In order to simplify the equations to be solved, further assumptions are made

p

0

$H(t)$ $H(t)$ $H(t)$

0 0 0 p

zero terms gives

$$\frac{\partial}{\partial t}\int_0^{H(t)} C\,dz - \frac{dH}{dt}C(H(t)) + \frac{\partial}{\partial x}\int_0^{H(t)}(uC)\,dz - \int_0^{H(t)}\frac{\partial}{\partial z}(v_pC)\,dz = 0. \quad (2.7)$$

Assuming that at (x,t) the sediment is distributed through a column of height y with a uniform concentration, i.e.

$$C = \begin{cases} C_0 & \text{if } 0 \le z \le y \\ 0 & \text{if } y < z \le H \end{cases} \quad (2.8)$$

implies

$$\frac{\partial C}{\partial z} = -C_0\delta(y) \quad (2.9)$$

where $\delta(y)$ is the Dirac delta distribution. The form of the concentration profile given by equation 2.8 also implies that at every point except $x = 0$ the concentration at the water surface is zero, so that for $x > 0$, $\frac{dH}{dt}C(H(t)) = 0$, which means that equation 2.7 becomes

$$\frac{\partial y}{\partial t} + \frac{\partial}{\partial x}(uy) + v_p = 0. \quad (2.10)$$

quation 2.10 is the continuity equation for the suspended sediment that will be used in this project to provide a numerical simulation of the deposition of sediment on a saltmarsh. Only values of $y \ge 0$ have any physical significance in the solution of this problem.

2.2.4 The Velocity Profile over the Saltmarsh

In order to solve equation 2.10 the functional form of the velocity over the saltmarsh is required. This can be obtained by solving an equation of continuity for the fluid derived from a consideration of mass balance in the fluid.

time, t, is defined such that $t = 0$ when the water instantaneously covers the marsh. The marsh will be covered by the tide whilst

$$0 < t < \frac{\pi}{\omega} - 2t_0. \tag{2.16}$$

2.2.6 The Model Equations

In summary, the following equation is used to model the transport of sediment over the salt marsh,

$$\frac{\partial y}{\partial t} + \frac{\partial}{\partial x}(uy) + v_p = 0 \tag{2.17}$$

with the boundary condition that

$$y(0,t) = H(t) \qquad \forall\ 0 \leq t \leq \frac{\pi}{2\omega} - t_0 \tag{2.18}$$

where the velocity and depth of water over the marsh are given by

$$u(x,t) = \frac{L-x}{H}\frac{dH}{dt} \tag{2.19}$$

$$H(t) = A\sin(\omega(t+t_0)) - b. \tag{2.20}$$

2.3 An Analytic Solution

The model equations given in Section 2.2.6 can be solved analytically for the case where $b = 0$, i.e. $H = A\sin(\omega t)$, by the method of characteristics described by Wood(1993).

From equations 2.19 and 2.20 we have that

$$u = (L-x)\omega\cot(\omega t) \tag{2.21}$$

and from equation 2.17 we have

$$u\frac{\partial y}{\partial x} + \frac{\partial y}{\partial t} = -\left(y\frac{\partial u}{\partial x} + v_p\right). \tag{2.22}$$

which implies from equation 2.21 that

$$\frac{dy}{dt} \quad (\omega \cot \omega t) y = \quad v_p. \tag{2.27}$$

Multiplying equation 2.27 by a factor $cosec(\omega t)$ gives

$$\frac{d}{dt} \quad \frac{y}{\sin(\omega t)} \quad = \quad \frac{v_p}{\sin(\omega t)}. \tag{2.28}$$

Given that, for $0 \quad t \quad \frac{\pi}{2}$ at $x = 0$, $\alpha = \omega t$ and $y = A \sin \alpha$, integration of equation 2.28 gives

$$y = A \sin(\omega t) \quad 1 \quad \frac{v_p}{A\omega} \ln \quad \frac{\tan \frac{\omega t}{2}}{\tan \frac{\alpha}{2}} \tag{2.29}$$

where α is given by equation 2.25.

The rate of deposition, $R(x, t)$, of sediment on the saltmarsh is given by

$$R = \begin{array}{ll} C_0 v_p & \text{if } y > 0 \\ 0 & \text{otherwise} \end{array} \tag{2.30}$$

so, in order to be able to determine the depth of sediment deposited on the marsh the time, $t_E(x)$, at which $y = 0$ is needed. The position, $x_E(t)$, at which $y = 0$ can be found from equation 2.29 by substituting in an expression for α from equation 2.25, giving

$$x_E(x) = L \quad 1 \quad \frac{\sin \; 2 \arctan \; e^{-\frac{A\omega}{v_p}} \tan(\frac{\omega t}{2})}{\sin(\omega t)} \quad . \tag{2.31}$$

Rearranging and using trigonometric identities gives

$$t_E(x) = \frac{2}{\omega} \arctan \quad \sqrt{\frac{[e^{\frac{A\omega}{v_p}}(1 \quad \frac{x}{L}) \quad 1]}{[1 \quad e^{-\frac{A\omega}{v_p}}(1 \quad \frac{x}{L})]}} \quad . \tag{2.32}$$

It must be noted that for $x > L[1 \quad e^{-\frac{A\omega}{v_p}}]$ the expression does not hold as there is no time for which $y = 0$ at these x values.

The tidal period over which equation 2.17 from section 2.2.6 is to be solved can be split into two distinct parts. The first is whilst the tide is rising and water, full of sediment from the channel, is coming into the region over the saltmarsh. The second part is after the tide has turned and the water, with some sediment remaining, equations numerically.

The saltmarsh is divided into equal sections of length $\Delta \ =$ with $_{j} = \ \Delta$. The time discretisation is done by using 2 equal time steps, $\Delta \ = (\frac{\pi}{2\omega} \quad _{0})$, with $_{n} = \ \Delta$. The solution $(\ _{j} \ \ _{n})$ is approximated by $^{n}_{j}$ and $[\ \ ]^{n}_{j} = \ (\ _{j} \ \ _{n}) \ ^{n}_{j}$. The depth of sediment deposited, $(\ _{j} \ \ _{n})$, on the marsh is approximated numerically by $^{n}_{j}$.

When the box scheme is applied to equation 2.17 the time derivatives are approximated by a weighted average of the finite difference form at two spatial points

$$— \quad \frac{(1 \qquad)}{\Delta}(\ ^{n+1}_{j} \qquad ^{n}_{j}) + \frac{}{\Delta}(\ ^{n+1}_{j+1} \qquad ^{n}_{j+1}) \qquad (3\ 1)$$

and the space derivatives are replaced by a weighted average of the finite difference forms at two time levels

$$—(\quad) \qquad \frac{}{} \qquad ^{n}_{j+1} \qquad ^{n}_{j} \qquad — \qquad ^{n+1}_{j+1} \qquad ^{n+1}_{j}$$

$$— \ \frac{^{2}}{^{2}} \qquad \frac{^{2}}{} \qquad ^{2} \qquad ^{2}$$

and

$$\frac{\Delta}{2}\frac{^2(\quad)}{^2} + \Delta \frac{^2(\quad)}{} + (\Delta \ ^2) + (\Delta \ ^2) \qquad (3\ 4)$$

respectively.

This gives a finite difference approximation to the continuity equation for the suspended sediment, equation 2.17 of the form

$$\frac{(1 \qquad)}{\Delta}(\ _j^{n+1} \qquad _j^n) + \frac{}{\Delta}(\ _{j+1}^{n+1} \qquad _{j+1}^n) +$$
$$\frac{(1 \qquad)}{\Delta}([\quad]_{j+1}^n \quad [\quad]_j^n) + \frac{}{\Delta}([\quad]_{j+1}^{n+1} \quad [\quad]_j^{n+1}) + \ _p = 0 \qquad (3.5)$$

with a truncation error, , given by

$$= \frac{\Delta}{2}\ \frac{^2}{^2} + 2\frac{^2(\quad)}{} + \frac{\Delta}{2}\ \frac{^2(\quad)}{^2} + 2\frac{^2}{} + (\Delta \ ^2) + (\Delta \ ^2) \quad (3\ 6)$$

quation 2.17 can be partially differentiated with respect to t giving

$$\frac{^2}{^2} = \frac{^2(\quad)}{} \qquad (3\ 7)$$

and with respect to x giving

$$\frac{^2(\quad)}{^2} = \frac{^2}{} \qquad (3\ 8)$$

Substituting these into equation 3.6 gives

$$= \frac{1}{2}(1 \quad 2\ )\Delta\ \frac{^2}{^2} + - \qquad \frac{^2}{^2} \qquad ^2 \qquad ^2$$

p

$$\overline{} \qquad \overline{}$$

$$^{n}_{j} \qquad ^{n}_{k} \; ^{ijk\Delta x}$$

$$\overline{}$$

$^{n}_{k} \; ^{ijk\Delta x}$

$$ikΔx \qquad ikΔx$$

$\frac{\Delta t}{\Delta x}$

$$\overline{} \qquad \overline{}$$

equation is linear in y, i.e. the velocity, u, is independent of y, the equation produced by discretising using the box scheme is linear and can be solved easily.

quation 2.17 is discretised using the box scheme with the parameters set at $= \; = 0\;5$. Rearranging the finite difference form, equation 3.5 to solve for $^{n+1}_{j+1}$ explicitly gives

$$_{j+1}^{n+1} = \frac{[\;_j^n \quad _j^{n+1} + \;_{j+1}^n]\Delta \quad + [[\quad]_j^n \quad [\quad]_{j+1}^n + [\quad]_j^{n+1}]\Delta \quad 2 \;_p\Delta\;\Delta}{[\Delta \quad + \quad (\;_{j+1} \quad _{n+1})\Delta\;]} \quad (3\;15)$$

This equation is then solved for 0 1 to step along the region from the seaward end for each time step 0 1, until the turn of the tide.

A numerical approximations ^n_E to $_E(\;_n)$ is obtained by using linear interpolation between the two points across which the sign of ^n_j changes. At each time level there will be a $_E$ such that $^n_{j_E}$ 0 and $^n_{j_E+1}$ xandF?48O 2Bxx2Bxdifferen5x0Fa5xthenF

$$^n_E \quad _E \quad \frac{^n_{j_E}}{^n_{j_E} \quad ^n_{j_E+1}}$$

^n_j

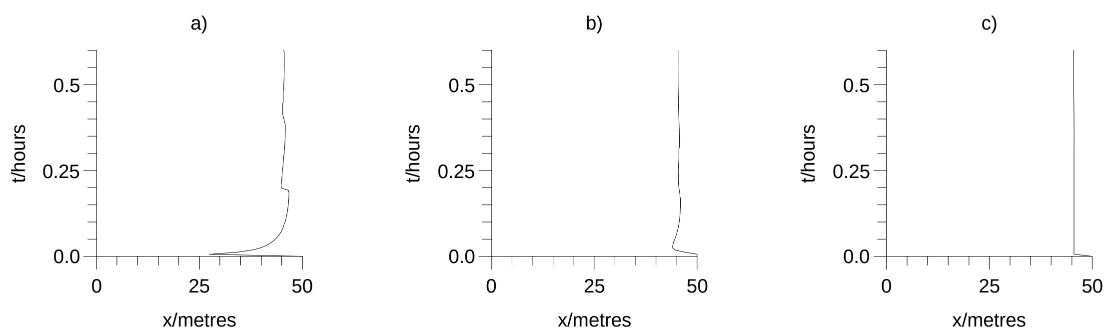
$$^{n+1}_j \quad ^n_j \quad ^n_j \quad \frac{(x_E^n - x_j)Cv_p\Delta t}{x_E^n - x_E^{n+1}} \quad ^p \quad _j \quad ^{n+1}_E \quad ^{n+1}_E \quad _j \quad ^n_E \quad ^n_j$$

^0_j

A nT 62 xpF? 62 xB64erecF e?4mff5 62 xtsds xB64ereeacF 62 B64eredealF? 6V2B4eree small the horizontal velocities are large compared to the settling velocities of the sediment. This means that for very small the column of sediment in the water is approximately the same depth, , as the water. Then, using the expression for the velocity from equation 2.19 the product , for small t, is given by

= ——— —— = ( )—— (3 18)

Both these expressions for were tried for the initial time step in the numerical scheme applied to the problem described in section 2.3 and the results compared



E p -4

$\frac{dH}{dt}$

ruled out.

$n{+}1$ $j{+}1$ n j $n{+}1$ j $n{+}1$ j $n{+}1$ j p

$j{+}1$ $n{+}1$

$\overset{n}{M}$

$\overset{n}{M}$

$\overset{n}{E}$ E

$M{-}1$

$\overset{n}{E}$

the position of ${}^{n}_{E}$ it is more important to have this value accurately evaluated than to have the height of the column at the end of the region accurately defined because only values of 0 have any physical significance in this problem and the value of ${}^{n}_{M}$ affects the solution at subsequent time levels only at ${}_{M}$ and therefore does not affect the solution in the interior of the region.

During the ebb tide the box scheme cannot be used quite as simply as during the flood tide. Since there is no boundary data no values are known at the new time level and it is therefore necessary to evaluate at least one of the heights, ${}^{n+1}_{j}$, at the new time level by some other method. If a position towards the landward end of the region is chosen then this new value can be used like a boundary condition and the box scheme can be used to step towards the seaward end from this point in the same way as it is used during the flood tide.

Several things need to be considered when calculating this 'boundary value', the most important of which is that in order to get a complete solution for the part of the marsh which still has sediment suspended above it, the first value at negative x direction and that u is negative this gives a scheme,

$$y_{j-1}^{n+1} = \frac{y_{j-1}^{n}\Delta x + ([uy]_{j-1}^{n} - [uy]_{j}^{n})\Delta t - v_p \Delta x \Delta t}{\Delta x} \tag{3.20}$$

There are two main problems with this method, firstly the explicit scheme given in equation 3.20 is only conditionally stable which will mean that $\Delta x, \Delta t$ will have to be varied to provide stability. Secondly, once a value has been obtained at the new time level a check will have to be made to ensure that the value is negative and if not a new $\Delta x, \Delta t$ chosen to get a negative value.

An alternative method is to trace along the characteristic from a point at the old time level, n, where the solution is non-positive to the new time level and evaluate a numerical solution at this new point. This method guarantees that the value at the new time level will be negative and does not create the same problems over choice of Δx and Δt. It does however require the characteristic equation 2.23 to be solved numerically, which may itself create problems with stability conditions.

The method used in this project is to trace the characteristic from the point where the numerical solution is zero, x_E^n to the next time level, $n+1$ by numerically solving the characteristic equation

$$\frac{dx}{dt} = \frac{L - x}{H}\frac{dH}{dt} \tag{3.21}$$

and to simultaneously update the values of y on this characteristic from the equation

$$\frac{dy}{dt} = -(y\frac{\partial u}{\partial x} + v_p) \tag{3.22}$$

by use of a numerical method. To avoid problems with stability the trapezium rule was chosen because it is unconditionally stable.

quations 3.21 and 3.22 are integrated from t_n to t_{n+1} using P time steps $\Delta t_c = \frac{\Delta t}{P}$. The position of the characteristic through x_E^n is given by $x_c(t)$ and is approximated by x_c^p at a time $t_p = t_n + p\Delta t_c$. The trapezium rule then gives

$$x_c^{p+1} = x_c^p + \frac{1}{2}\Delta t_c \;\; u(x_c^{p+1}, t_{p+1}) + u(x_c^p, t_p) \qquad (3.23)$$

This equation is solved iteratively by making an initial guess

$$x_c^{p+1,(0)} = x_c^p + \Delta t_c u(x_c^p, t_p) \qquad (3.24)$$

and then using the iteration

$$x_c^{p+1,(k+1)} = x_c^p + \frac{1}{2}\Delta t_c \;\; u(x_c^{p+1,(k)}, t_{p+1}) + u(x_c^p, t_p) \qquad (3.25)$$

until two consecutive approximations differ by less than a specified tolerance.

If the value of y on the characteristic is approximated by y_c^p then application of the trapezium rule to equation 3.22 gives

$$y_c^{p+1} = \frac{y_c^p \quad v_p + \frac{1}{2}y_c^p u_x(x_c^p, t_p) \;\; \Delta t_c}{1 + \frac{1}{2}\Delta t_c u_x(x_c^{p+1}, t_{p+1})} \qquad (3.26)$$

With x_c^P and y_c^P known it is possible to use the box scheme to integrate along the region towards the seaward end. The first step must be done with a steplength defined by $\Delta x' = x_c^P \quad x_{j_c}$ where j_c is such that $_{j_c} \quad {}_c^P \quad {}_{j_c+1}$. This means that an approximation to $({}_c^P \quad {}_n)$ is needed also. The value of $({}_c^P \quad {}_n)$ is approximated by $_{c'}$ using linear interpolation between the two calculated values either side of it. If $_c({}_{n+1}) = {}_E({}_n)$ then the interpolation is done between ${}_{j_c}^n$ and ${}_E^n$, otherwise ${}_{j_c}^n$ and ${}_{j_c+1}^n$ are used.

The box scheme applied to the first horizontal step at each time level becomes

$${}_{j_c}^{n+1} = \frac{[\ {}_{j_c}^n + \ {}_{c'} \quad {}_c^P]\Delta \ ' + [[\quad]_{j_c}^n \quad ({}_c^P \quad {}_n) \ {}_{c'} \quad ({}_c^P \quad {}_{n+1}) \ {}_c^P]\Delta \quad 2 \ {}_p\Delta \ \Delta \ '}{[\Delta \ ' \quad ({}_{j_c} \quad {}_{n+1})\Delta \ ]}$$

(3 27)

For subsequent horizontal time steps the box scheme is given by

$$y_{j-1}^{n+1} = \frac{[y_{j-1}^n + y_j^n - y_j^{n+1}]\Delta x + [[uy]_{j-1}^n - [uy]_j^n - [uy]_j^{n+1}]\Delta t - 2v_p \Delta t \Delta x}{(\Delta x - u(x_{j-1}, t_{n+1})\Delta t)}. \tag{3.28}$$

If $y(0, t_{n+1}) > 0$ then the position of x_E^{n+1} and the depth of deposited sediment are evaluated in the same way as for the flood tide.

22

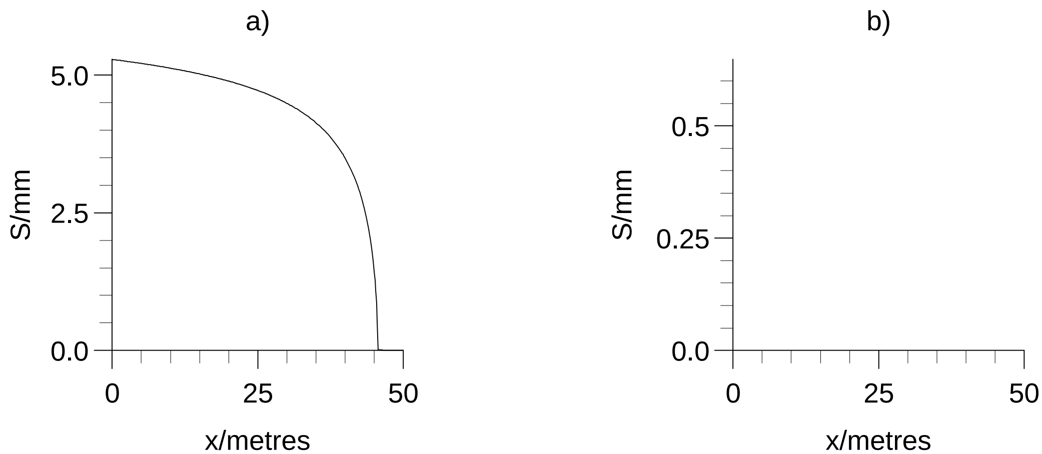
If the characteristic being traced goes out of the region, i.e. $x_c \le 0$ then the integration of the characteristic equation stops, the time at which $x_c = 0$ and the value of y_c are approximated using linear interpolation between t_p and t_{p+1}. In this case or if $y_0^{n+1} < 0$ then the method evaluates the time, t_s, at which $y(0,t) = 0$ by using linear interpolation along the time axis at $x = 0$, updates the deposited sediment for the shortened time step, $t_s - t_n$ and stops.

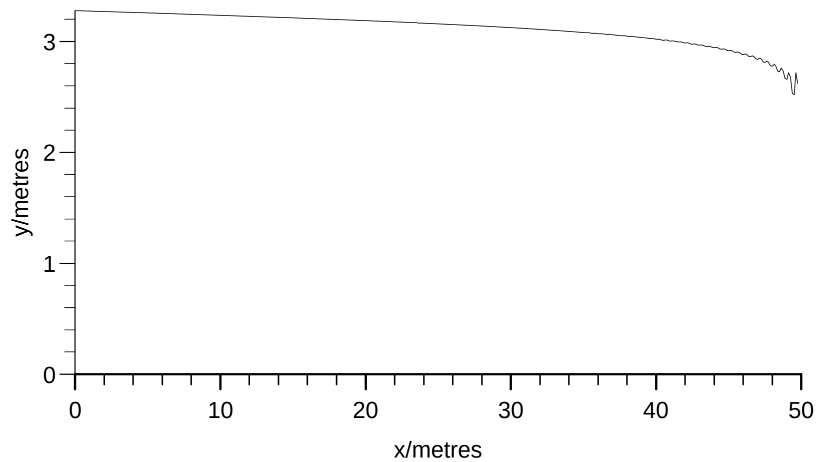
If the numerical solution continues until $t = \frac{\pi}{\omega} - 2t_0$ without giving $y_0^{n+1} < 0$ then the method forces $y_0^{2N} = 0$ and interpolates between x_E^{2N-1} and zero to evaluate the deposition for the final time step.

The numerical method was tested by using it to solve the problem described in section 2.3 and comparing the results with the analytic solution given there.

The testing was done with the parameters of the problem set as; $= 5$, $=$

$_p$ $^{-4}$ $^{-1}$ $_p$ $^{-5}$ $^{-1}$


a)
b)
5.0
2.5
0.0
S/mm
0
25
50
x/metres
0.5
0.25
0.0
S/mm
0
25
50
x/metres



y/metres
3
2
1
0
0
10
20
30
40
50
x/metres

N	M	rror, E_s/mm	
		$v_p = 3 \times 10^{-4} ms^{-1}$	$v_p = 3 \times 10^{-5} ms^{-1}$
50	50	1.9657×10^{-3}	7.4086×10^{-4}
50	100	1.1740×10^{-3}	4.6134×10^{-4}
50	200	8.1780×10^{-4}	2.9941×10^{-4}
50	400	5.9296×10^{-4}	2.0041×10^{-4}
50	800	4.4190×10^{-4}	1.3744×10^{-4}
100	50	2.0946×10^{-3}	6.4394×10^{-4}
100	100	7.2483×10^{-4}	3.5656×10^{-4}
100	200	4.4250×10^{-4}	2.0693×10^{-4}
100	400	3.0412×10^{-4}	1.2698×10^{-4}
100	800	2.1305×10^{-4}	7.9361×10^{-5}
200	50	2.3575×10^{-3}	6.4530×10^{-4}
200	100	7.0094×10^{-4}	3.4750×10^{-4}
200	200	2.6464×10^{-4}	1.8575×10^{-4}
200	400	1.5609×10^{-4}	1.0268×10^{-4}
200	800	1.0442×10^{-4}	5.8160×10^{-5}
400	50	2.3432×10^{-3}	6.7564×10^{-4}
400	100	7.2911×10^{-4}	3.5625×10^{-4}
400	200	2.6778×10^{-4}	1.8822×10^{-4}
400	400	9.2843×10^{-5}	9.9209×10^{-5}
400	800	5.3609×10^{-5}	5.2819×10^{-5}
800	50	2.1998×10^{-3}	6.8404×10^{-4}
800	100	7.3507×10^{-4}	3.6360×10^{-4}
800	200	2.3386×10^{-4}	1.9122×10^{-4}
800	400	7.6897×10^{-5}	9.9516×10^{-5}
800	800	3.0227×10^{-5}	5.2401×10^{-5}

Table 4.1: rror in the numerical solution of the deposited sediment with $A = 5m$, $T = 12hrs$ and $C = C_0$ for $v_p = 3 \times 10^{-4} ms^{-1}$ and $v_p = 3 \times 10^{-5} ms^{-1}$ for varying values of N and M.

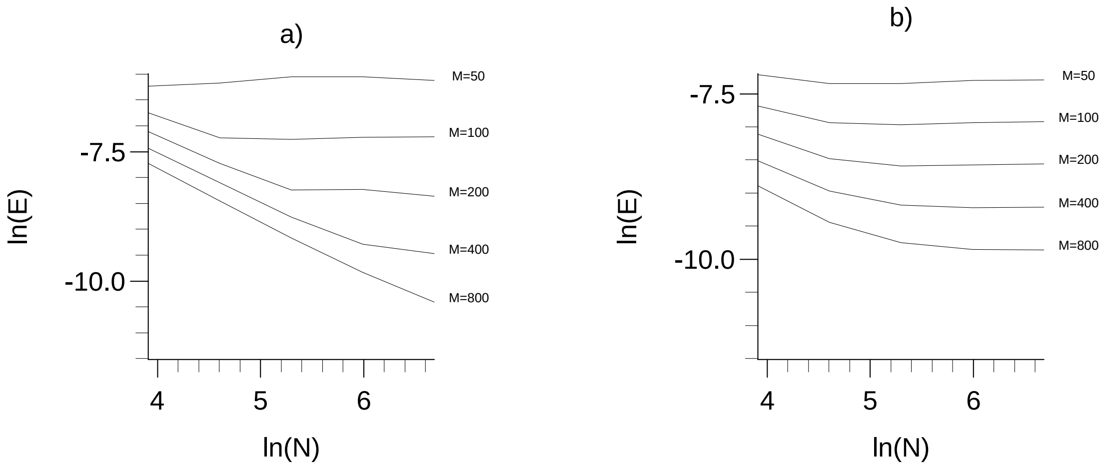


Figure 4.3: The variation of the error, $_s$, with for fixed values of , with a) $_p = 3 \quad 10^{-4} \quad ^{-1}$ and b) $_p = 3 \quad 10^{-5} \quad ^{-1}$.

can be seen that for fixed the error can not be reduced greatly, if at all, by increasing the value of beyond that of . Secondly, for a fixed value of the error improves if is increased, even beyond the value of , although this

s

discretisation. Similarly to estimate the order of accuracy in space the data for $N = 800$ was used. The estimate was made by evaluating the gradient of the line of best fit through the data points using the method of least squares.

For $N = 800$ the gradient, r_M, of the line of best fit for $\ln(E_s)$ against $\ln(M)$ is $r_M = -1.5628$ for $v_p = 3\times10^{-4}ms^{-1}$ and $r_M = -0.9282$ for $v_p = 3\times10^{-5}ms^{-1}$.

For $M = 800$ the gradient, r_N, of the line of best fit for $\ln(E_s)$ against $\ln(N)$ is $r_N = -0.9730$ for $v_p = 3\times10^{-4}ms^{-1}$, the data for $v_p = 3\times10^{-5}ms^{-1}$ is not suitable for a straight line approximation and so was not used.

These results show that the scheme is at least first order in time and space, i.e.

$$E_s \approx O(\frac{1}{N}) + O(\frac{1}{M}) \tag{4.2}$$

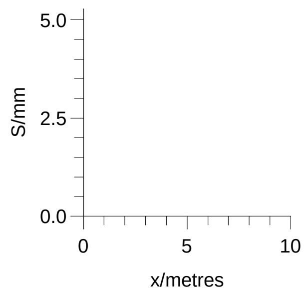
$$= O(\Delta t) + O(\Delta x) \tag{4.3}$$

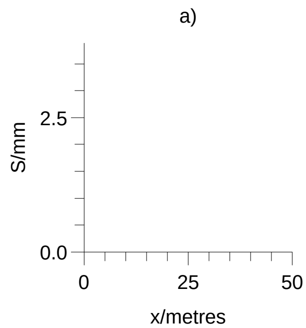
The scheme used is first order because although the second order box scheme is used to solve for y, linear interpolation which is only first order is used to calculate the approximations to x_E and hence the scheme becomes first order overall.

4.2 Experiments

The model was used to examine how the depth of sediment deposited on the marsh varies with the parameters A, b, v_p, L for a fixed tidal period $T = 12hrs$.

a)




a)
S/mm
2.5
0.0
0
25
50
x/metres

with the boundary condition

$$y'(0,t') = H'(t') \qquad 0 \quad t' \quad \frac{\pi}{2} \quad t'_0 \tag{4.14}$$

where

$$u'(x',t') = \frac{1 \quad x'}{H'}\frac{dH'}{dt'} \tag{4.15}$$

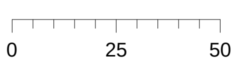
$$H'(t') = sin(t' + t'_0) \quad b' \tag{4.16}$$

From this non-dimensional form of the equation it can be seen that the solution is dependent on two non-dimensional parameters, and $_p$.

By holding , , and constant and varying $_p$ shows how the depth of sediment deposited on the marsh depends on the settling velocity. From figure 4.6 it can be seen that the sediment with small settling velocities is uniformly distributed over the saltmarsh and that as the settling velocity increases a gradient in the depth of sediment deposited develops across the marsh and the deposited sediment does not extend across the whole width of the marsh.

Figure 4.7 shows that increasing the ratio reduces the depth of sediment deposited on the marsh for a given concentration and also causes sediment of a given settling velocity to be less evenly distributed over the marsh.

Sediment suspended in tidal water is not made up of particles of a unique size and settling velocity but particles of many different sizes. This model can be used to investigate this case because it has been assumed that each particle acts independently and therefore the solution for each particle size is independent of the others.


0
25
50

Ch pter 5

Conclusions

A simple one-dimensional mathematical model of sediment transport over a saltmarsh was developed from a generalised three-dimensional mass balance equation for suspended sediment by making a series of simplifying approximations regarding the flow of the water over the marsh, the suspension of sediment in the water and the topography of the marsh. ssentially, the marsh was assumed to be flat and uniform in the along channel direction, the flow was assumed to be non-turbulent and hence the diffusion of sediment in the fluid was ignored and the sediment was assumed to be distributed throughout a column of water with a uniform concentration. The height of this column of water was used as the variable to describe the quantity of sediment suspended above the marsh and an equation governing its variation in space and time was obtained by depth integration of the simplified mass balance equation. An analytical solution of this equation was obtained for a saltmarsh at mean sea-level to compare with the solution obtained from the numerical model.

The model equation was solved numerically by use of the box scheme [Preiss-

p

p

p

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