o t to p op t n su t rn t v nor s roup n s H n propos
u oup sto st vo ut on r o or n v u s v our tr tor s t n
n ross poss su roup or t on sso t t t rn t v so nor s nov t
o t s o s n t t ns on r t t n t o op t n t vo v n so
n t or n t poss t o n t or n u nst t s t n t n v u s t v tor
n tor v our st t n s

In s t on ntro u t not on o t v tor n tor s st t t o sts t n ot
t ps o o n t t t r tur s In s t on pr s nt t o n
n n s s o ts n s s o t t o n t on o ur n nst t pro ss
qu t o on n t v tor n tor n s n t o o

v r t o s p p ss v— oup n t n t os n v u s st s u ur n nst t s n tur r t un or s tr r su t n n o su s ts o r t v n r s t v t on n r t v n r s n t on s o urs n t oup n t n n v u s—s stron r or t n tor v r s t n or t t v tor v r s u nst t s s ount r ntu t v t rst s t s n n v u s r s n p r s n nt t or p t ss p t on . ut on o n t ons o s tr s to v op us n p r s oup n o t v tor n tor v r s s pot nt n s or r t n v r t ross popu t ons

It s t us r son to postu t t t n t t t .

o ons r n vo ut on qu t on or **A**(t) E s ssu to vo v n p n nt t ou s on t on p n nt on t urr nt n t or so s on t on on r t urr nt su stru tur s orr t ov r t o r t r t n o u pro t str ut on or utur n t or vo ut on on t on on ts urr nt stru tur s

$$P_{\delta t}(\mathbf{A}(t+\delta t)|\mathbf{A}(t)),$$

t s nou to sp ts p t v u $E(\mathbf{A}(t+\delta t)|\mathbf{A}(t))$ tr ont n n pro t s ro s n r t n p n nt r qu v n s tr v s n $E(\mathbf{A}(t+\delta t)|\mathbf{A}(t))$ $_B$ $\mathbf{B}P_{\delta t}(\mathbf{B}|\mathbf{A}(t))$, n

$$P_{\delta t}(\mathbf{B}|\mathbf{A}(t)) \prod_{i=1, i'=i+1}^{N-1,N} (\mathbf{W}_{i,i'})^{(\mathbf{B})_{i,i'}} (1-\mathbf{W}_{i,i'})^{1-(\mathbf{B})_{i,i'}},$$

r $\mathbf{W}$ $E(\mathbf{A}(t+\delta t)|\mathbf{A}(t))$ H n s sp our o or t sto st n t or vo ut on v

$$E(\mathbf{A}(t+\delta t)|\mathbf{A}(t)) \quad \mathbf{A}(t) + \delta t F(\mathbf{A}(t), X(t)),$$

v s $\delta t \to$ H r t r tr v u un t on $\mathbf{F}$ s s tr : t s ro on n su t t nts o r t n s r n r t

$$F(\mathbf{A}(t), X(t)) \quad -\mathbf{A}(t) \circ \{\text{Death rates}\} + (1-\mathbf{A}(t)) \circ \{\text{Birth rates}\}.$$

tr s po u ons r t on r I t r r no st n u s n v u s t n t n ust nv r nt un r n p r ut t on o t v rt s o or p $\mathbf{Q}$ s n $\mathbf{N}\times\mathbf{N}$ p r ut t on tr t n ust v

$$F(\mathbf{A}, X) \quad \mathbf{Q}^T F(\mathbf{Q}\mathbf{A}\mathbf{Q}^T, \mathbf{Q}X)\mathbf{Q}, \quad \text{or} \quad \mathbf{A}, X,$$

s s not s r str t v s t pp rs For s nvo v n po no s or H r pro u ts o po no s n $\mathbf{A}$ s t s t s r str t on In GH t s s tr su st r t o n ppro s

p n n o $\mathbf{F}$ upon X s r t s n t u oup s t s st v n n o op t s nt on n t ntroo s n F s r rFF J F surn t s r t t n t orr spon n p rs o v rt st t s o s t n ons r t n n r o s n n t t $\mathbf{m}$ n $\mathbf{x}$ $(\mathbf{x}_1, \mathbf{x}_2)^{\mathbf{T}}$ or no It s v n

$$\mathbf{f}(\mathbf{x}) \quad (\mathbf{p} - \mathbf{x}_1\mathbf{x}_2^2, \mathbf{q} - \mathbf{x}_2 \cdot \mathbf{x}_1\mathbf{x}_2^2)^{\mathbf{T}},$$

r $\mathbf{p}$ n $\mathbf{q}$ r pos t v onst nts It s t t qu r u

$$\mathbf{x}^* \quad \frac{\mathbf{p}}{(\mathbf{p} \cdot \mathbf{q})^2}, \mathbf{p} \cdot \mathbf{q} \quad .$$

Cons r t $\mathbf{D}$ $(\mathbf{d}_1, \mathbf{d}_2)$ v

$$\mathbf{df}(\mathbf{x}^*) \quad \begin{matrix} -(\mathbf{p} \cdot \mathbf{q})^2 & -\frac{2\mathbf{p}}{\mathbf{p}+\mathbf{q}} \\ (\mathbf{p} \cdot \mathbf{q})^2 & \frac{\mathbf{p}-\mathbf{q}}{\mathbf{p}+\mathbf{q}} \end{matrix} \quad .$$

s s st t tr n on

$$(\mathbf{p} \cdot \mathbf{q})^3 > (\mathbf{p} - \mathbf{q}) .$$

o ons r $(\mathbf{df}(\mathbf{x}^*) - \boldsymbol{\lambda}_{\mathbf{i}}\mathbf{D})$ Its nv u s s $\boldsymbol{\sigma}_{\pm}$ s t s

$$\boldsymbol{\sigma}^2 - \boldsymbol{\sigma}(-(\mathbf{p} \cdot \mathbf{q})^2 \cdot \frac{\mathbf{p} - \mathbf{q}}{\mathbf{p} \cdot \mathbf{q}} - \boldsymbol{\lambda}_{\mathbf{i}}(\mathbf{d}_1 \cdot \mathbf{d}_2)) \cdot \mathbf{h}(\boldsymbol{\lambda}_{\mathbf{i}})$$

r $\mathbf{h}(\boldsymbol{\lambda})$ $\mathbf{d}_1\mathbf{d}_2\boldsymbol{\lambda}^2 - \boldsymbol{\lambda}(\mathbf{d}_1\frac{\mathbf{p}-\mathbf{q}}{\mathbf{p}+\mathbf{q}} - \mathbf{d}_2(\mathbf{p} \cdot \mathbf{q})^2) \cdot (\mathbf{p} \cdot \mathbf{q})^2$ ot s n $\boldsymbol{\lambda}_{\mathbf{i}} >$ v $\boldsymbol{\sigma}_+ \cdot \boldsymbol{\sigma}_- <$ n n t r s oss n st t on n $\mathbf{h}(\boldsymbol{\lambda}_{\mathbf{i}}) <$ s o urs n on $\boldsymbol{\lambda}_{\mathbf{i}} \in (\boldsymbol{\lambda}_-, \boldsymbol{\lambda}_+)$ s r

$$\boldsymbol{\lambda}_{\pm} \quad \mathbf{d}_1\frac{\mathbf{p}-\mathbf{q}}{\mathbf{p}+\mathbf{q}} - \mathbf{d}_2(\mathbf{p} \cdot \mathbf{q})^2 \pm$$

5. Projections

H r s pr s nt so s u t ons o t s st n t output s v r on s r s o v r r tr s t s v s r u t to v su s p o so r nt pro t ons to ptur t stru tur o ur n t or t t st p u t t to s o t t t s ps u o p r o

ur pro t ons r o t or

$$\mathbf{A} \to \mathbf{P_B}.\mathbf{A} \quad \mathbf{A} \bullet \mathbf{B}\ \mathbf{B}$$

r not s t Fro n us nn r pro u t v n $\mathbf{A} \bullet \mathbf{B}$ $_{\mathbf{i,j}}$ $\mathbf{A_{ij} B_{ij}}$, $\mathbf{A}$ s n n tr t n t n $\mathbf{B}$ s v n r s tr tr nor so t t $\mathbf{B} \bullet \mathbf{B}$ sp nn n t pro t on sp s $\mathbf{A} \bullet \mathbf{B}$ s t p tu o t pro t on $\mathbf{P_B}$

o or sp not

$$1 \quad \frac{}{\mathbf{n.n-}} 1,$$

t nor qu tr n n **clique projection** not $\mathbf{R}$ t

$$\mathbf{R.A} \quad \mathbf{P_{\hat{1}}.A}.$$

p tu o t pro t on $\mathbf{R}$ s p ounts t ns t n t n r n tr $\mathbf{A}$

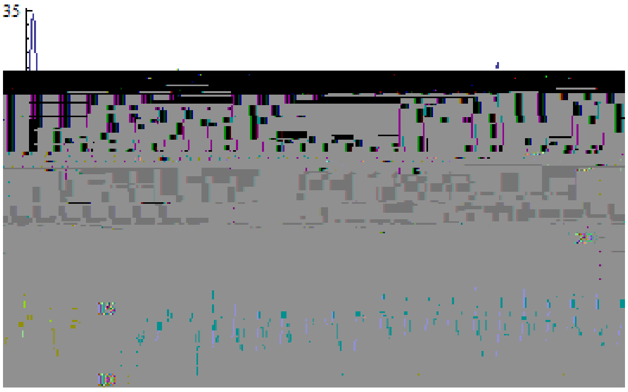
n r t n vo v n n t or $\mathbf{A.t}$ n st t s $\mathbf{X.t}$ us n s s r t t st ps $\boldsymbol{\delta}\mathbf{t}$ n t p r t r v u s $\mathbf{N}$ $\mathbf{p}$ $\mathbf{q}$ $\mathbf{d}_1$ $\mathbf{d}_2$ $\boldsymbol{\delta}$ n $\boldsymbol{\omega}$. In t s s st rt ro n Er os n n t or t ns t t ts n t t st p r n s n nt urn n p r o or n t n pro t ons t t F ur s o s t pro t on p tu p ott nst t n r ustr t s t ost p r o t n t s n t or s ns t

A ost p r o v our n usu o s rv n o p n s st s pro t n t n s nto t o ns ons s n st t sp n o oo or s on pro t on t t s ort o on to t rst

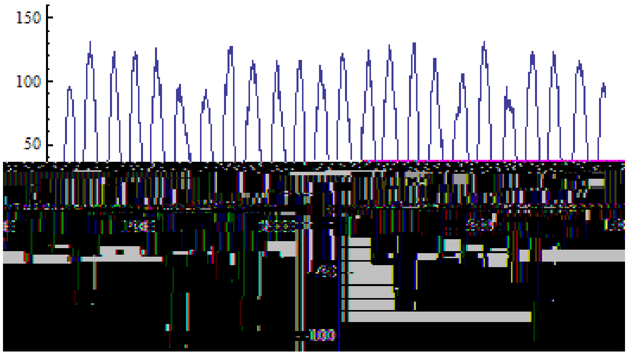
uppos t t $\mathbf{W}$ s r s tr n su t t $\mathbf{W} \bullet \mathbf{W}$ n $\mathbf{W} \bullet 1$ s n our s on pro t on $\mathbf{S}$ t $\mathbf{S.A}$ $\mathbf{P_W.A}$ so t t 1 n $1 \bullet \mathbf{W}$

At n t $\mathbf{t}$ $\mathbf{A.t}$ o pos nto o n t on o our pro t ons $\mathbf{S}$ n $\mathbf{R}$ p us rror t r s not $\mathbf{E.t}$ s o o s

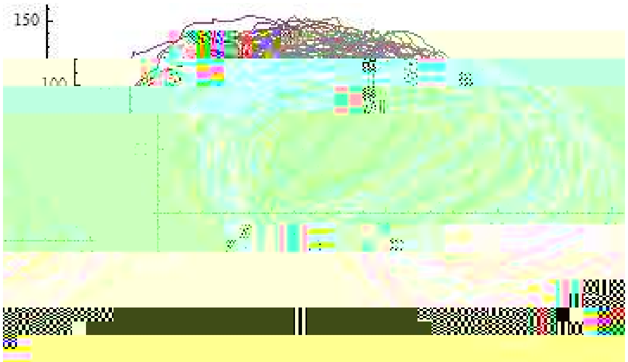
$$\mathbf{A.t} \quad \mathbf{R.A.t} \quad \mathbf{S.A.t} - \mathbf{E.t}$$



F r p Q pro on **R** p o n ro p n n p r o n r Q n or pro on pp o n r o n n or on r p r **N** **p** **q** **d**$_1$



F r . p Q pro on **S** p o n ro p n n p r o n r Q n or pro on pp o n r o n n or on r p r **N** **p** **q** $\mathbf{d}_1$ $\mathbf{d}_2$. $\boldsymbol{\delta}$. h $\boldsymbol{\omega}$.



F r . pro on p p o n o r p p or pro on **R** on h p or pro on **S** on p n s nor nv tors tt n $\delta t \to 0$ o t n t t r n st s st

$$\rho \quad -\rho \quad -\varphi \|u\| \quad \omega \quad -\rho\, \varphi \|u\| ,$$
$$u \quad df\, x^* \quad -\rho N D\, u.$$

For p s t $\varphi \|u\|$ $-$ t n $\frac{\|u\|-}{\mu}$ / r , $\mu > 0$ os n so t t φ s t s r p s s r s $\|u\|$ n r s s t rou n su t v u s nu r so ut on s o s t t t s st s st s poss ss s p r o or ts

6. Conclusion

s o s s o t t n n v u s o r n n qu r u t n t r t v t on n n tor t n n s r oup p r s n o op t n s ou p t r t v o so onv r n to t nor n ons rv t v v ours n o st ross popu t on n r n onst nt st t o u t ros op s tu t on s pr t t ourn s or n v u s r not t n ot t r n st n sto st v rs ons o t o r r r so o nt tors n so o ono s o ss rt t t v r nt tt tu s s n so nor s r qu r rs n r pos on popu t ons: or s t r r v n p rt p r n s n v nts But r n s t t t tr ns nt st n o o ust r su roups o n v rs v s n n r nt v our t n u oup s st s s n t nor st t o rs t n so t s v n t out t rn t s n or n t r s

Ev n t sto st n s or $A\, t$ r r p t r n st n s or t o un t on n tr on o t ns s st t t ts p r o n r n n so s ns t v p n n s t rn t v o s s uss n . In su s s t or ts r ot no t t t os t ut n not pr t t r n sp n v u s o r t v n t or r t v t v t t n utur s

Acknowledgements

Gr n ro H D J n rsons C B st t t rou tr osur Int rn t t t s

rson t ov n n Coo J B r s o t n Ho op n so n t or s Ann v o

Hoo B I us on H rp rCo ns

B n ro t J Gr CA J nss n E n rs A u ontro o urr nt st tus n utur r t ons J s r Jun

Gr J A ps o o o r n str ss C r C r n v rs t r ss

Gr n ro tt rns n v s n E t on or

r J A n Gr n ro Ap r o n s n t r n st o o u tur ss n t on Dr t su tt