Early Warning with Calibrated and Sharper Probabilistic Forecasts

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Abstract

Given a nonlinear deterministic model, a density forecast i

ar na ca brat on w. . n not a . o at t. sa t D ns t or asts ar sa to b probab st a a brat an on t. I s ar un or str but arF na a brat on r rs to t. as w. n t. t av raF o a ns t or asts s qua to t. at o a or asts . r s no p r a wa o ass ss nF x an a brat on g

Gn t nF *tra* t. n on tur t. at w. n a subs t o t. s o o a . brat on . o s t. n t. pr t v str but ons ar at ast as spr a out as t. a or asts w. . on tur t. t r a *s arpn ss pr nc p* t r *s arpn ss* s s to . av b n o n b Bross an t s a asur o . ow on ntrat probab st or asts ar an a prop rt o o boF to t s attr but s w m ato oF s u u m o s o a brat on ass ss ar probab st an m arF na a brat on w t m p as s t at ntrop s ou b us to m asur s arpn ss n w propos t on t at a r ss s Gn t nF *a* s on tur os xp a n t n nFs r as Bro r m t s r t m to not n u nF a n un t ons n t ns t st m at on was ts Fuarant n r as n var an o t pr t v str but on o par to t raw ns m b w m onstrat t at t us o var an to m asur s arpn ss ou b m s a nF

s pap r s orFan s as o ows n xt s t on s uss s or ast qua t s t at ar m u at v m asur b t oFar t m s or nF ru In part u ar a m o pos t on o t s s or nF ru s pr s nt s arpn ss pr n p on tur b Gn t nF *a* s s uss an a r vant propos t on pr s nt n § In § t m t o o oF m p o to pro u r st or asts s out n su ts on rn nF ns t or asts obta n v a t oFar t m s or nF ru w t r sp t to a non n ar tron r u t ar s uss n § t on F v s on u nF r m ar s App n s A an B onta n t proo o t propos t on on rn nF t s arpn ss pr n p an app n x C onta ns proo s or t r st o t propos t ons

2 Probabilistic-Forecast Quality

o m ssp at on p a s m tat ons on t va u o probab st or asts n t ot r an onst m rs o r asts m a m an or t w str but ons t at ar bot *ca* *rat* an *s arp* I su or asts ar ssu at onF t or ons t n ar warn nF s a or suFF st t at t s qua t s an b m u at quant b t oFar t m s or nF ru propos b Goo r ar ot r s ornF ru s ava ab or s t ons s Gn t nF a t r For nstan t r s t Br r s on Br r s ow v r m pos s nto m an t r s urp s o o w ar not r vant to our s uss on an t s su tab or at Fr a v nts A F n ra sat on o t Br r s or to ns t or asts s t

o two Gauss ans w av ow r ntrop t an a s nF Gauss an str but on o t s var an u or a str but on o a F r var an an av a ow r ntrop t an t at o ow r var an

arp ss as a so b n quant b on n nt rva s a t r a Gn t nF a Con n nt rva s s ar a s ar w a n ss to var an n t s ns t at a b o a str but on t at s a r on ntrat on t two o s an av ar F r on n nt rva s t an a un o a str but on t at s a r spr a out A so F v n two non s tr str but ons w o t s s arp r ou p n on w at t on n v s

2.3 Calibration

a brat on o ns t or asts s a w tro n sub t u o t t ratur ta s t stan t at a a brat or ast nF s st tanta ount to a orr t sp Corra wanson prov a o pr ns v surv o o a stat st a t n qu s or ass ss nF a brat on o ns t or asts to t n t un r nF o s orr t so wor o Gn t nF a str s a s or b prov nF a a brat on ra wor t at a o at s o ssp at on bro own a brat on nto t r o s a o w ou b ass ss s parat uppos a probab t or ast nF s st ssu s pr t v str but ons $\{\ _t\ x\}_{t=1}^{T}$ w t ata F n rat nF pro ss ssu s a or asts $\{\ _t\ x\}_{t=1}^{T}$ Gn t nF a t n n t o ow nF o s o a brat on

- s qu n $\ _t\ x\}_{t=1}^{T}$ s *probab s a y ca brat* r at v to $\{\ _t\ x\}_{t=1}^{T}$

$$\sum_{t=1}^{T} \ _t\{\ _t^{-1}\ p\}\quad p\quad p$$

- s qu n $\{\ _t\ x\}_{t=1}^{T}$ s *xc an a brat* r at v t $\{\ _t\ x\}_{t=1}^{T}$

$$\sum_{t=1}^{T}\ _t^{-1}\{\ _t\ x\}\quad x\quad x$$

- or ast r s *ar na y ca brat*

$$\frac{}{T}\sum_{t=1}^{T}\ _t\ x\qquad \frac{}{T}\sum_{t=1}^{T}\ _t\ x$$

I w av a t s r s o obs rvations t n $\ _t\quad x_t$ s a *probab y nt a trans or* I Corra wanson D o a n o t o t I s s qu va nt to probab st a brat on Gn t nF a A v sua nsp t on o I sto ra s wou r v a obv ous partur s ro un or t un r nF o s orr t sp an on $\ _t$

uppos w av a t s r s o ns t or asts $\{ _t x \}_{t\,1}$ n n t or ast r s ato oF as

$$ _T\ x \quad \frac{}{} \sum_{t=1}^{T} \ _t\ x $$

n a or ast r w o ssu s

$$ _t\ x \quad \overline{\ }_T\ x \quad \frac{}{} \sum_{t=1}^{T} \ _t\ x $$

or a $\{$ $\}$ to b t *n t c ato o ca or cast* I $_t\ x$ $_T$ $\overline{\ }_T\ x$ t n w av t *c ato o ca or cast* A or ast r s *n t ar na y ca brat* $_T\ x$ $\overline{\ }_T\ x$

For a pra t a purpos s s n t an w av no a ss to t $_t\ x$ s H n t s u t to ass ss n t arF na a brat on I $_T$ $_T$ $\approx$ w r s s tr t n w an ta to b arF nouF to va uat arF na a brat on o t s n w an us t H nF r stan o ar an o put

$$ _T\ \ _c \quad \frac{}{} $$

Gndddd.6

w an or ns t or ast st at s o t or

$$^{(t)}\ x \quad \sum_{i=1}^{N} \left\{ \left(x - X_i^{(t)} - \right. \right\}$$

w r an ar r sp t v ban w t an o s t para t rs os n a or nF to past r or an an s t rn un t on ns t or ast n rs ro t tra t ona ar n st at s b t o s t para t r It s s ar to t Ba s an o Av raF propos b a t r a w a m or bas orr t on an qua w F ts H r t ns b m b rs ar x anF ab an o not r pr s nt st n t o s t nF us nF v r an o s not a ount or m o m ssp at on

o a ount or m o m ssp at on t us rst not a r or o past t s r s an orr spon nF ns or asts b V_T $\{ \ _t X^{(t)} \}_{t=1}^{T}$ n t ns t or asts w os para t rs an a s t b a nF nto a ount past p r or m an m a b not b $^{(t)}\ x|V_T$ $^{(t)}\ x|V_T$ as t sa or as n ts para t rs ar s t b o nF t m n m sat on

$$\min_{\sigma>0,\mu} \ \sum_{t=1}^{T} \ \mathrm{oF}\ ^{(t)} \ _t|V_T$$

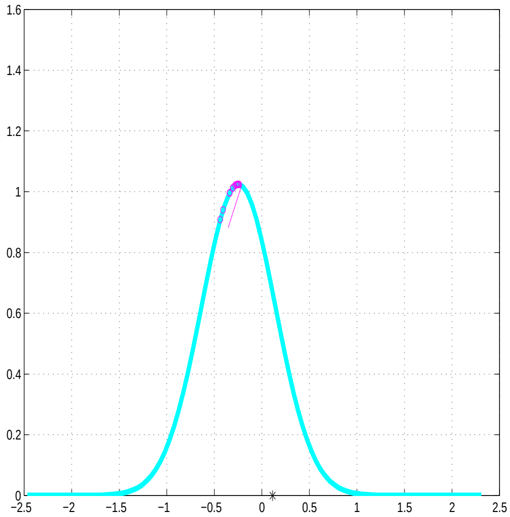
v n r rta n assu pt ons o nF t m n m sat on n s an t para t rs c an c

w n g Disp t t s r u t on so m m xtur or asts m a st b ss s arp t an m ato oF n t s ns o ntrop A stra F t orwar app at on o t Ku ba L b r an J ns n s n qua t s a s to t r at ons

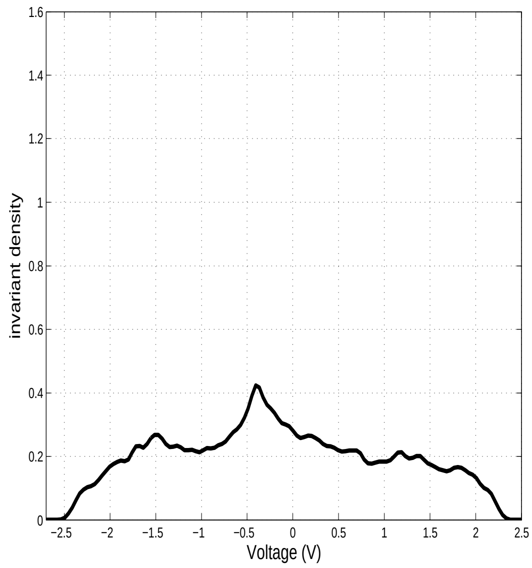
$$\{ (t) \} - c \leq \{ (t) \} \leq \{ (t) \} - \{ (t) c \} - \{ c (t) \} - c$$

w r $-\int x$ oF x x an $-\int x$ oF x x ar t ntrop an ross ntrop r sp t v r or t n ssar an su nt on t ons or $\{ (t) \} \geq c$ to o ar t at c $\{ (t) c \}$ an c $\{ (t) \}$ r sp t v n v r t m ato o s s arp r t an t m xtur or ast t s ou b ssu as t or ast nst a o t m xtur

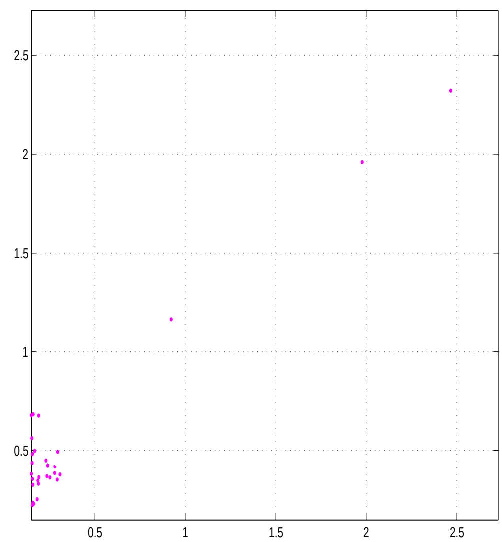
It s not obv ous w at t t o t m xtur s on a brat on x pt t at n u nF m ato oF m prov s t KL stan r m t a or asts v r ss t m xtur para m t r t at m n m s s t oF ar t m s or s t quat on

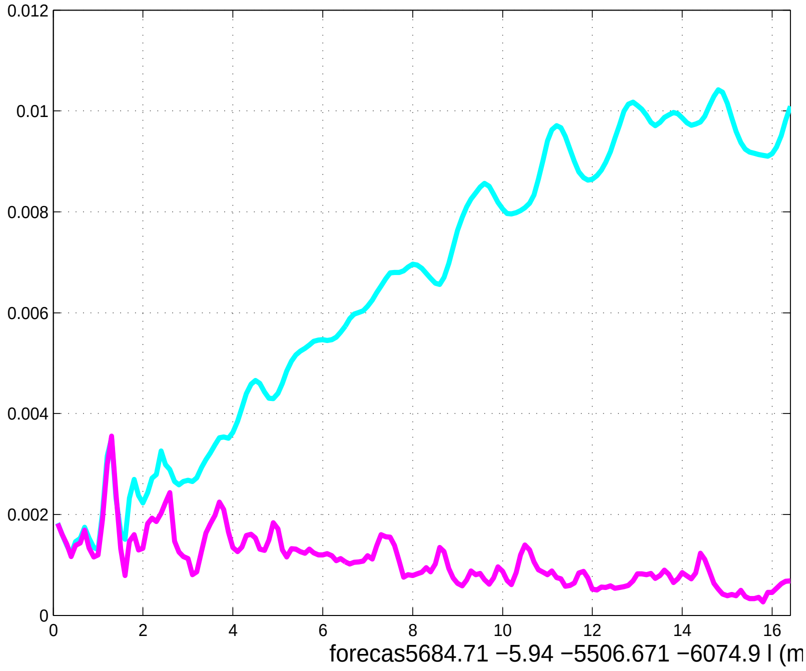


1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
−2.5
−2
−1.5
−1
−0.5
0
0.5
1
1.5
2
2.5

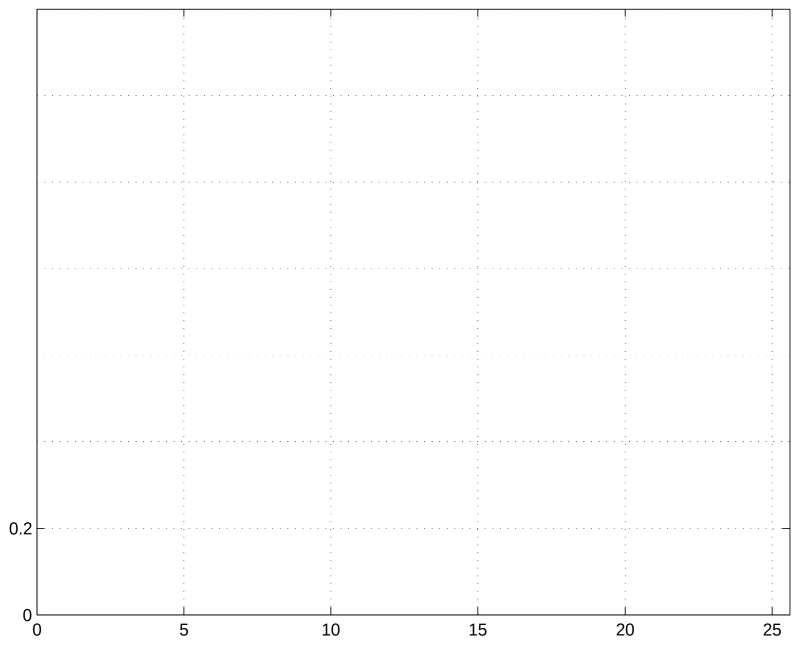


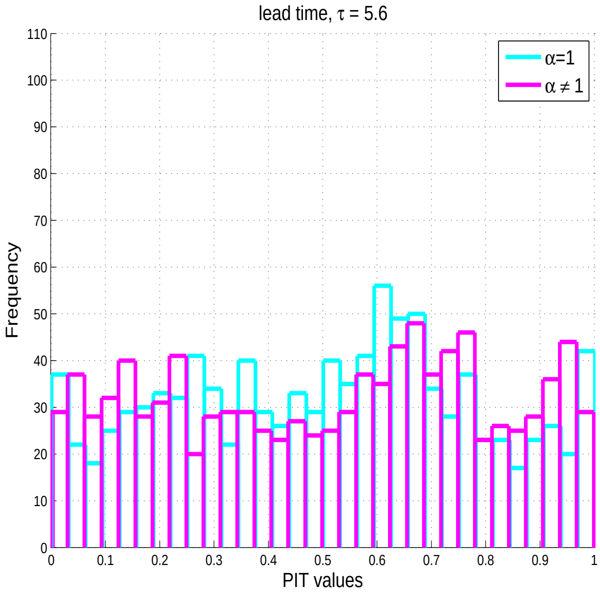
F Fur *Graph of the climatology of the circuit estimated from data. Its entropy is 2.15, which is greater than the entropies shown in figure 1.*

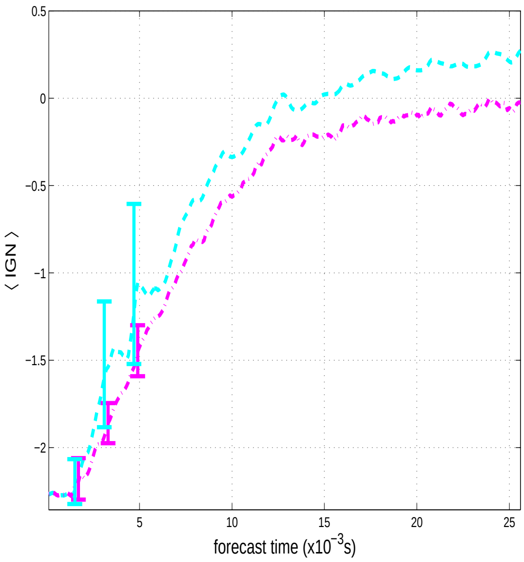




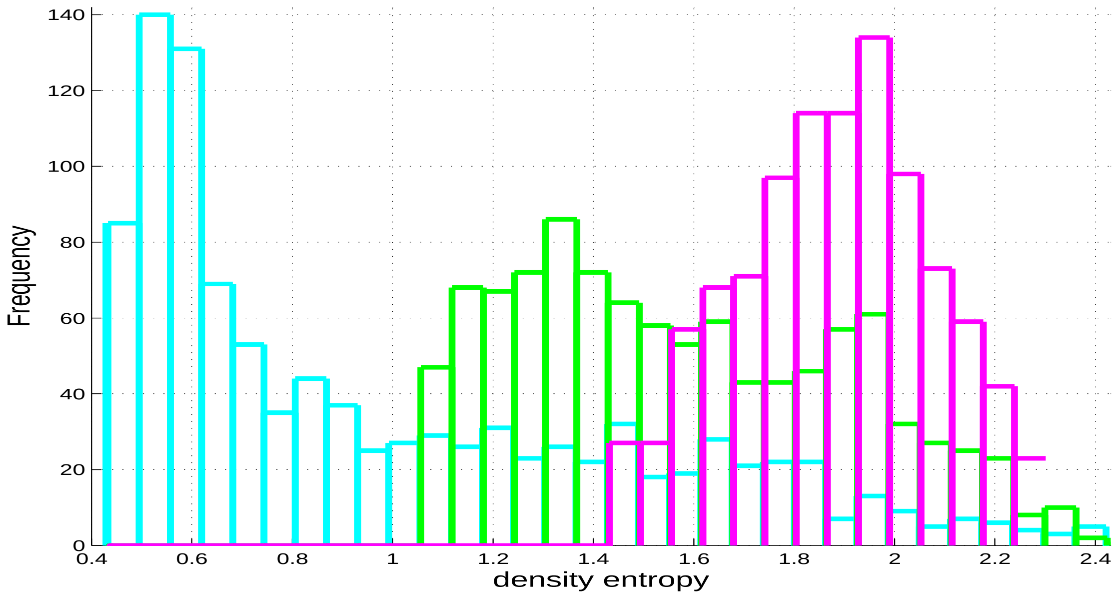
0.012
0.01
0.008
0.006
0.004
0.002
0
0
2
4
6
8
10
12
14
16
forecas5684.71 −5.94 −5506.671 −6074.9 l (m


0.2
0
0
5
10
15
20
25


lead time, τ = 5.6
Frequency
PIT values
α=1
α ≠ 1
0
10
20
30
40
50
60
70
80
90
100
110
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



0.5
0
−0.5
−1
−1.5
−2
⟨ IGN ⟩
5
10
15
20
25
forecast time (x10^{-3}s)


Frequency
density entropy
0
20
40
60
80
100
120
140
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4

annon CE tor *Co un cat on n t pr s nc o no s* vo u ro

$_{\mathbf{t}}\,x \qquad {}_{\mathbf{t}}^{-1} \quad {}_{1}\{\ {}_{1}\ x\ \}$ It t. n o ows t. at

$$= \sum_{\mathbf{t}=1}^{\mathbf{T}} \ {}_{\mathbf{t}} \left[\ {}_{\mathbf{t}} \left\{\ {}_{\mathbf{s}}^{-1}\ x_{\mathbf{s}}\ \right\}\right]$$

t r m a b at ast anot r p □ an p su t at