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Flow-Dependent Balance Conditions for Incremental Data Assimilation: Elliptic Operators

by

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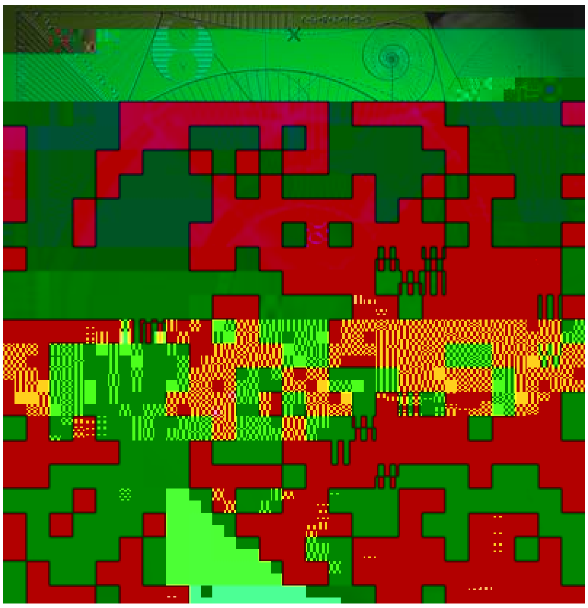

b (L *op. cit*

et al., 1 2)

(1).

2 B B

2.1, 2.2, 02. 2.

2.1 A B

et al. (2000).

$(u;v;p;\ ) \mapsto (\ ;\ ;{}^{A}p)$,

$$\equiv \cdot r \times r^2 ; \quad (1\ )$$

$$\equiv r \cdot r^2 ; \quad (1b)$$

3.2

$$u_g = -\frac{g}{af}\frac{\partial h}{\partial}; \qquad v_g = \frac{g}{af}\frac{\partial h}{\partial}: \qquad (1\ \ )$$

fi ^{b},

$$^{b} \equiv - \frac{}{f}(\cdot \nabla)(\times);$$

fi

$$u^{b} \equiv u_g - \frac{}{f}\left(\frac{u_g}{a}\frac{\partial v_g}{\partial} + \frac{v_g}{a}\frac{\partial v_g}{\partial} + \frac{}{a}u_g^2\right); \qquad (1\ \ )$$

$$v^{b} \equiv v_g + \frac{}{f}\left(\frac{u_g}{a}\frac{\partial u_g}{\partial} + \frac{v_g}{a}\frac{\partial u_g}{\partial} - \frac{}{a}u_g v_g\right); \qquad (1\ \ )$$

, (12). (1) (1) (1),

$$\begin{aligned} u^{b} = &-\frac{g}{af}\frac{\partial h}{\partial} - \frac{g^2}{a^3 f^3 \ ^2}\left(\frac{\partial h}{\partial}\frac{\partial^2 h}{\partial \ \partial} - \frac{\partial h}{\partial}\frac{\partial^2 h}{\partial \ ^2}\right. \\ &+ \left(\left(\frac{\partial h}{\partial}\right)^2 + \ ^2 \left(\frac{\partial h}{\partial}\right)^2\right) \\ &- \left.\frac{}{f}\left(\frac{\partial h}{\partial}\right)^2\right); \end{aligned} \qquad (1\ \ )$$

$$\begin{aligned} v^{b} = &\frac{g}{af}\frac{\partial h}{\partial} + \frac{g^2}{a^3 f^3}\left(\frac{\partial h}{\partial}\frac{\partial^2 h}{\partial \ \partial}\right. \\ &- \left.\frac{\partial h}{\partial}\frac{\partial^2 h}{\partial \ ^2} + \frac{\partial h}{\partial}\frac{\partial h}{\partial} + \frac{}{f}\frac{\partial h}{\partial}\frac{\partial h}{\partial}\right); \end{aligned} \qquad (1\ \ )$$

$$\equiv \frac{df}{d}: \qquad (1\ \ )$$

fi

h

$$
\begin{aligned}
{}_{b} \;\; & \frac{g}{f} r^{2} h + \frac{g}{f^{2} a^{2}} \frac{@h}{@} + \frac{2g^{2}}{f^{3} a^{4} \quad {}^{2}} \Bigg(\bigg(\frac{@^{2} h}{@ \; @} \bigg)^{2} \\
- \;\; & \frac{@^{2} h}{@ \;\; {}^{2}} \frac{@^{2} h}{@ \;\; {}^{2}} + 2 \qquad \frac{@h}{@} \frac{@^{2} h}{@ \; @} + \qquad {}^{2} \; \bigg(\frac{@h}{@} \bigg)^{2} \\
+ \;\; & \qquad\qquad \frac{@h}{@} \frac{@^{2} h}{@ \;\; {}^{2}} \\
+ \;\; & \frac{1}{2} \Bigg(\bigg(\frac{@h}{@} \bigg)^{2} + \qquad {}^{2} \; \bigg(\frac{@h}{@} \bigg)^{2} \Bigg) \\
+ \;\; & \frac{}{f} \Bigg(2 \frac{@h}{@} \frac{@^{2} h}{@ \;\; {}^{2}} - 2 \frac{@h}{@} \frac{@^{2} h}{@ \; @} - 2 \qquad \bigg(\frac{@h}{@} \bigg)^{2} \\
+ \;\; & 2 \frac{}{f} \bigg(\frac{@h}{@} \bigg)^{2} - \frac{}{2} \qquad\quad \bigg(\frac{@h}{@} \bigg)^{2} \Bigg) \\
- \;\; & \frac{1}{2f} \frac{@}{@} \; \bigg(\frac{@h}{@} \bigg)^{2} \Bigg) :
\end{aligned}
\qquad (1 \;\;)
$$

(1) h

$$\frac{@^2 h}{@ \quad ^2}; \qquad \frac{@^2 h}{@ \quad ^2}; \qquad \frac{@^2 h}{@ \quad @}: \tag{22}$$

ffi A, B, C, D E (21) w (; ; $h; p; q$), p $@h{=}@$; q $@h{=}@$.

fi (1) (20)

, fi 1 > 0,
(20) 0 .
, (20), ffi
(1), fi A.
A, (20)

$$\frac{(1+2\ )f}{2}+\frac{2}{fa^2}\left(v_g^2+u_g^2\right)-\frac{}{fa}u_g+\frac{2}{f^2a^2}u_g^2$$
$$+\left(\frac{2\ \ 2}{f^3a^2}-\frac{@}{@}\frac{2\ \ 2}{f^2a^2}\right)v_g^2>\ hq^b\ \ (f+\ ^b):$$

f
v .
v . v h
, .
fi fi , v
h , , $@h=@$ 0. w
h . / 10 1 (. (2)-10)-2 ( )
, (.)-2 ( )- 0 (
(.)- ( )- -
v w (1) (20), w
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4 .

(.) fi
fi , ^{b}
(1) 20).
.

h^0

$$b_0 \equiv \frac{g}{f}\, r^2 h^0 + \frac{g}{f^2 a^2}\frac{\partial h^0}{\partial} + \frac{2g^2}{a^4 f^3}\,{}^2 \left(2\frac{\partial^2 \bar{h}}{\partial\ \partial}\frac{\partial^2 h^0}{\partial\ \partial}\ {}^2\ 2 h \partial\right.$$

$$- \frac{\partial^2 \bar{h}}{\partial\ {}^2}\frac{\partial^2 h^0}{\partial\ {}^2} - \frac{\partial^2 \bar{h}}{\partial\ {}^2}\frac{\partial^2 h^0}{\partial\ {}^2} + 2\ \frac{\partial^2 \bar{h}}{\partial\ \partial}\frac{\partial h^0}{\partial}$$

$$+ 2\ \frac{\partial \bar{h}}{\partial}\frac{\partial^2 h^0}{\partial\ \partial} + 2\ {}^2\ \frac{\partial \bar{h}}{\partial}\frac{\partial h^0}{\partial} + {}^2\ \frac{\partial \bar{h}}{\partial}\ {}^0$$

∂h
∂

4.2

5 A

v v . (0) (1) fi . v ff fl ffi ff ffi ' 2- , ff -v (*et al.*, 1 2). ffi ' - (1). -v fi

$$h = \frac{1}{g}\left(gh_0 + a^2 A(\ ) + a^2 B(\ ) \quad R + a^2 C(\ ) \quad 2R\ \right);$$

$$u = a! \quad + aK \quad ^{R-1} \left(R \ ^2 - \ ^2\right) \times \quad R\ ;$$

$$v = -aKR \quad ^{R-1} \quad R\ ;$$

$$= 2! \quad - K \quad ^{R} \left(R^2 + \ R + 2\right) \times \quad R\ ;$$

h_0 $A(\ ); B(\ )$ $C(\ )$ w

$$A(\ ) = \frac{!}{2}(2\Omega + !) \quad ^2 + \frac{1}{\ }K^2 \quad ^{2R} \quad (R+1) \times \quad ^2 + \left(2R^2 - R - 2\right) - 2R^2 \quad ^{-2} \ ];$$

$$B(\ ) = \frac{2(\Omega + !)K}{(R+1)(R+2)} \quad ^{R} \left[\left(R^2 + 2R + 2\right) - (R+1)^2 \quad ^2\right];$$

$$C(\ ) = \frac{K^2}{\ } \quad ^{2R} \left[(R+1) \quad ^2 - (R+2)\right];$$

$!$, K R -v v , h_0. R v , , v v (, 1). h_0, v , $!$ fl K v . v fl ff , B_u. v

$$B_u \equiv \frac{\sqrt{gh}}{fL} = \frac{L_R}{L};\qquad (\quad)$$

L L_R -

.

ff v B_u ff .

(2002).

fi , 1 , $h_0 =$ 0m ! =

$K =$

b ‾ ‾ ffi B^2

.

ffi (s^{-6})	1	2	
$\dfrac{1\ g^2 \left(\frac{@\bar{u}_g}{@}\right)^2}{\bar{h}^2}$	$2{:}\ \times 10^{-14}$	$:\ 1 \times 10^{-16}$	$:\ \times 10^{-18}$
$-\dfrac{2g^2\ \ \frac{@\bar{u}_g}{@}\bar{v}_g}{a\bar{h}^2_2}$	$-2{:}2\ \times 10^{-14}$	$-\ :\ 2 \times 10^{-16}$	$-\ :\ 2 \times 10^{18}$
$\dfrac{1\ g^2\ \ {}^2\ \bar{v}_g^2}{a^2\bar{h}^2}$	$:1\ \times 10^{-15}$	$1{:}\ \times 10^{-16}$	$1{:}\ 2 \times 10^{-18}$
B^2	$:\ \times 10^{-15}$	$1{:}\ 2 \times 10^{-16}$	$:\ \times 10^{-18}$

, 1 10^2 ffi AC . v , 2 10 . w 2 L ff b .

h_{11} / :11 10 . 1 0 2(`) .1 3: :11 () / 1 10 . -. 0 (1 -2

ffi AC

ffi (s^{-6})		1	2
$\frac{g^2 f^2}{\bar{h}^2}$	2	: $\times 10^{-11}$	2: $\times 10^{-14}$	:11 $\times 10^{-14}$
$\frac{g^2 f}{\bar{h}}$	$\frac{@\bar{u}_g}{@}$	h_{11}	0

8 g 2 @ @813(i.56.15 6.74 Td[(@)]TJ /F8 99 T22)1.1388 0 Td4.882u4[(8960[(2674 Tf 5.75[.875TJ /F8 9.963 Tf 6.1363 T60[(-40 Td6.0

ffi (ms^{-3}) 1 2

ffi (ms^{-3})		1	2	
gf	$h_{\lambda\lambda}$	1.01×10^{-3}	1.01×10^{-3}	1.01×10^{-3}
fg	h	$:0 \times 10^{-4}$	$:0 \times 10^{-4}$	$:0 \times 10^{-4}$
$^2 fg$	$h_{\theta\theta}$	$:0 \times 10^{-4}$	$:0 \times 10^{-4}$	$:0 \times 10^{-4}$
$2g\frac{\partial \bar{u}_g}{\partial}$	$h_{\theta\lambda}$	$2: 2 \times 10^{-5}$	2.2×10^{-5}	[illegible] $\times 10^{-5}$
$2g \partial \bar{v}_g h$		$:0 \times 10^{-5}$	$2.0 \times$ [illegible]	[illegible]

.74 7io.P d[( )]TJ /F15 6.974 Tf 6.23 0 Td[(5)]TJ

v ,

(1) (2002).

(1), v

(1) v

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