School of Mathematics, Meteorology
and Physics

Department of Mathematics

Preprint MPS_2010_15

21 April 2010

State estimation using model order reduction for unstable systems

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State estimation using model order reduction for unstable systems

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T b a a cc a a ca fl d fl .
F a , d c a ab a ca a fi d
b b a a a . T fi d
b a a a a a b a b d b c
d a ca c a . U a d a b a a -
a Ga -N d d d c a
a ab . I a a d d
d a a b ba d a c d d d
d c d ab . T a ca ,

a ca . *Da a a a* c a fi d b a b
c b b a a da a a ca d . I
c -d a a a a da a a (4D-Va)
a b d a a a a b

$$\min_{x} \phi(x) = f(x)^T f(x); \qquad (1)$$

c $f(x)$ c d a ca d T
d b a a a a a Ga -N d,
a a a c a 4D-Va [1, 2]. W a d
a c a d d Jac b a c $f(x)$ a d
c Jac b a a ca d c d
a a a d (TLM). S fica ca
d c d a d a d ab b a .
I ab TLM a d a ca d
a ab a fi a T a c
b ab a a ab d b d .

A c d a ac d c c b
(1) a a TLM b a a d d a a a
. W ad a a a ac ca c

I ac c , a a a b (1) f a d fi d (2) ca b d b a Ga -N d. T a a a a ac a (k) a a a c

$$\phi(\delta x_0^{(k)}) = \|J_f \delta x_0^{(k)} + f(x_0^{(k)})\|_2^2; \tag{3}$$

J_f d Jac b a f. T a d fi d a $x_0^{(k+1)} = x_0^{(k)} + \delta x_0^{(k)}$. I (2) a Jac b a J_f b

$$J_f = \left[\left(B_0^{\frac{1}{2}}\right)^T \quad \left(R_0^{\frac{1}{2}} H_0\right)^T \quad \left(R_1^{\frac{1}{2}} H_1 M_{1;0}\right)^T \quad ::: \quad \left(R_N^{\frac{1}{2}} H_N M_{N;0}\right)^T \right]^T; \tag{4}$$

a d b a a d d a c $H_i := \frac{@h_i}{@x_i}(x^($ $x^($

F a a d (k) d . W
c d a - a a a a

$$\mathcal{S} : \begin{cases} \delta x_{i+1} &= M\delta x_i; \\ d_i &= H\delta x_i; \end{cases} \tag{6}$$

a (5), c a a c M a d H a a -
a - a a $M_{i+1;i}$ a d H_i d
$[t_0; t_N]$. T a a $\delta x_0 = B_0^{\frac{1}{2}}!$ a a d b d a d a -
ab a a d c a a c a $B_0 \in \mathbb{R}^{n\ \ n}$, $! \sim \mathcal{N}(0; I)$.
T a c T (6) a c a a d
c d c b b a c d a . I
d fi d a

$$T : \mathbb{C} \;\to\; \mathbb{R}^{p\ \ m}; \tag{7}$$

$$z \;\mapsto\; T(z) := H(zI - M)\ \ ^{1}B_0^{\frac{1}{2}}: \tag{8}$$

T (6) a ab , . . a
a M a d a a d c c . T b ab fi d
d a a (6) d a ab d d c c
ab . T d α-b d d ba c d ca a
c b d d [6] α-*b* *ded* , . . . a
a a M a c c a d a
ad α. F a ab $\mathcal{S}$ b fi d
c a α. T α-b d d ba c d ca c c
a d a a c U a d V, c , c a c d
,

$$\mathcal{S} : \begin{cases} \delta\hat{x}_{i+1} &= U^T M V \delta\hat{x}_i; \\ \hat{d}_i &= H V \delta\hat{x}_i; \end{cases} \tag{9}$$

d a c $\delta\hat{x}_i = U^T\delta x_i \in \mathbb{R}^r$, $r \ll n$, a a
d $\mathcal{S}$ M c , a ba b d
c $h_{1;}$ - [6]:

$$\|T - \hat{T}\|_{h_{\infty,\alpha}} \leq 2(\sigma^{(}$$

$F : \mathbb{C} \to \mathbb{R}^{m \quad p}$ a c c c c a d
ad α a d, , α-b d d . T ca
$\sigma^{(\ )}_{r+1}; : : : ; \sigma^{(\ )}_{n}$ a Ha a α-ca d $\mathcal{S}$,
b a c

$$T \quad = \frac{1}{\alpha}H(zI - \frac{1}{\alpha}M) \quad {}^{1}B_0^{\frac{1}{2}};$$

d Ga -N c d . A a d c d a
d a Ead d $x-z$ a d ba c c
ab , c d a c a a
d d .

4.1. E e e a de g

T d a a 2D Ead d [7] a d -
c b d. T ba c a b a a a d a , z,
a d a b d a b da , $z=\pm\frac{1}{2}$.

F [8] a d a a - c a
c (QGPV) . T a a b ba
b a c , $b=b(x;z;t)$, b da , $z=\pm\frac{1}{2}$, a $t=0$. T
d ca c d ba c a c-
, = $(x;z;t)$, c a fi :

$$\frac{@^2}{@x^2}+\frac{@^2}{@z^2}=0; \qquad z\in[-\frac{1}{2};\frac{1}{2}];\ x\in[0;X]; \tag{13}$$

b da c d

$$\frac{@}{@z}=b; \qquad z=\pm\frac{1}{2};\ x\in[0;X]: \tag{14}$$

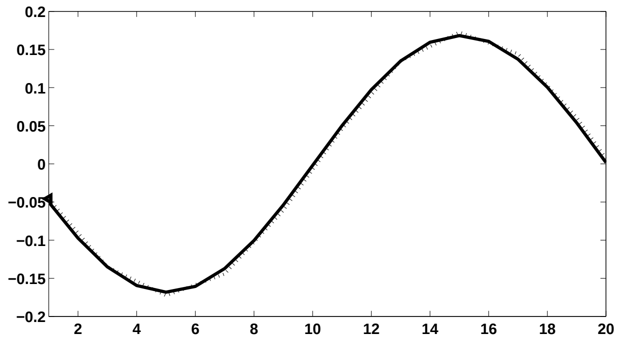
P ba ba c a a ad c d a b ba c a fl
a d c b d b -d a QG d a c a :

$$\left(\frac{@}{@t}+z\frac{@}{@x}\right)b=\frac{@}{@x}; \qquad z=\pm\frac{1}{2};\ x\in[0;X]: \tag{15}$$

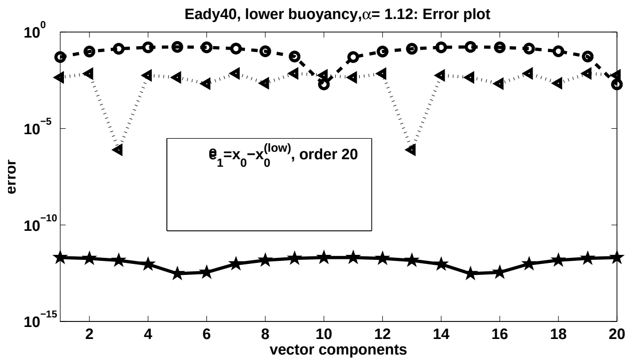
T a a b da c d x-d c a a b d c
c a a a , t, a d , z, $b(0;z;t)=b(X;z;t)$ a d $(0;z;t)=$
$(X;z;t)$. A [8] d a x, z, a d t.

I a d , Ead d d c d 11
ca a c . T a 20 d a
40 d d a c b. T ad c a
a d c d a a ad c c . W [8]
d a . T b a a H c c a b a a
a a c .

d c d 20, c c a a
b ab .



(a) Solution on lower boundary



(b) Error on lower boundary

Figure 1: Comparison of low resolution (dotted line with triangles), standard balanced truncation (dashtione

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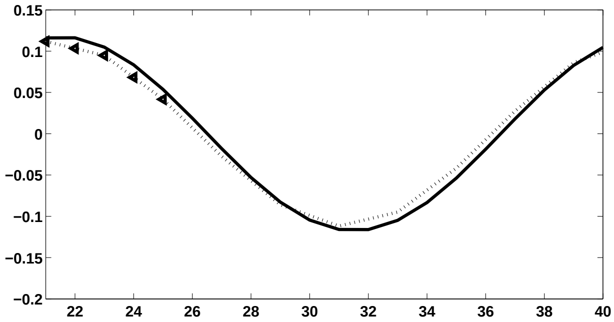

(a) Solution on upper boundary

(b) Error on upper boundary

Figure 2: Comparison of low resolution (dotted line with triangles), standard balanced truncation (dashed line with circles) and α-bounded balanced truncation (solid line with stars) approximations to the buoyancy on the upper boundary

T a ca α-b d d a - a c . T b fi ca b d a b a c d c d d a . W α-b d d d b a c fica a a ca d a d a da d ba c d ca d F 3(a) a a a d (c) F 3(b), 4(a) a d 4(b) a d ff d a a . T α-b d d a a d a c a a a a , d d a a d c c (F 3(b),

c c). I c a , a da d ba c d ca d (F 4(a),
c c) ca ab a c a d

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