Boundary integral methods in high frequency scattering

[illegible]

Abstract

In this article we review recent progress on the design, analysis and implementation of numerical-asymptotic boundary integral methods for the computation of frequency-domain acoustic scattering in a homogeneous unbounded medium by a bounded obstacle. The main aim of the methods is to allow computation of scattering at arbitrarily high frequency with finite computational resources.

[illegible]

n o cc cy o **N** d **L/λ**

nd ∂u/∂n o d n d nd d o nd y n od o o
o c o y ∂u/∂n y c o yno By con **hybrid methods** c
d c n o o n c on oy y o c n y o o n n y c n o on
o o c on n ∂u/∂n n o on n o d d c y n n c
od on y o y y n co on n o d nd y d od c
o ncy nc
o o o n y do y d o n do
y nd y n o o **S** **D** **D′** nd **H** d n d c y y

$$\mathbf{S}\psi \quad \int_{\Gamma} \quad x, y \;\psi\; y \; ds \; y \;, \qquad \mathbf{D}\psi \quad \int_{\Gamma} \frac{\partial \quad x, y}{\partial n \; y} \psi \; y \; ds \; y$$

$$\mathbf{D}'\psi \quad \int_{\Gamma} \frac{\partial \quad x, y}{\partial n \; x} \psi \; y \; ds \; y \;, \qquad \mathbf{H}\psi \quad \int_{\Gamma} \frac{\partial^2 \quad x, y}{\partial n \; x \; \partial n \; y} \psi \; y \; ds \; y \;.$$

c on c n n n

$$\mathbf{A}'v \quad \mathbf{I} \quad \mathbf{D}' - \eta \mathbf{S} \; v \quad f \;, \qquad v \quad \partial u/\partial n \;.$$

n on o on no n nd d oy c]] d] d

on cond nd n on c d n n no n o on
$\mathbf{v}$ $\frac{}{n}$ nd on on o n o nd y
c n y o $\mathbf{C}^1$ c n n o o $\mathbf{D}$ nd $\mathbf{S}$ n co c
on nd d nc on c o $\mathbf{A}$ co c on d n y o o
n c c n d on o y on c n o nd d n c
c n n nd co oc on od n c o yno nc on
d o n y d n d n c o on $\mathbf{v}_N$ y n o o
o o

$$\|\mathbf{v} - \mathbf{v}_N\| \le C \inf_{N \in \mathbf{S}_N} \|\mathbf{v} - \varphi_N\|,$$

$\mathbf{S}_N$ d no n d n on on c n d nd $\mathbf{N}$ d
c on d n on c $\mathbf{S}_N$ Mo c y o o y
d n d n od nd co oc on od c c n A n on
$\mathbf{C} >$ nd $\mathbf{N}_0 >$ c od o $\mathbf{N} \ge \mathbf{N}_0$
§ o o d

B d on on c n n o n c n y o od o c n
o n c on d on

d n o ood $\mathbf{k}$ d nd n n d n on o on c $\mathbf{S}_N$ o
o on o $\inf_{N \in \mathbf{S}_N} \|\mathbf{v} - \varphi_N\|$ o n o y o
$\mathbf{k} \to \infty$ c no y d nd on $\mathbf{k}$ nd o d no $\mathbf{S}_N$

oo o o d nd nc o y con n $\mathbf{C}$ n on
$\mathbf{k}$ o y o n n nd c o nd dn o d o $\mathbf{k} \to \infty$

d n o ood od o n n n c od n o
o on c n d y o n co on
c n o nd d $\mathbf{k} \to \infty$

o n d n $\mathbf{v}$ o on o o
on o o d n con n o c d $\mathbf{N}$ $\mathbf{k} \to \infty$ c n on n y
o o n n §

o c c o n y o cond nd n on
od o c n y $\mathbf{N}$ $\mathbf{N} \ge \mathbf{N}_0$ o c n $\mathbf{k}$
non n y n d n o o o $\mathbf{A}$ n y no c n
n o on on o o d $\mathbf{N}$ con n $\mathbf{C}$ d nd on $\mathbf{k}$ o
o o d $\mathbf{C}$N

o o o con n **B** nd α n on n y o Mo

n on od o o on d o n **any** n d n on

c $\mathbf{S}_N$ $\mathbf{L}_2$ $\mathbf{v}_N$ $\mathbf{S}_N$ c

$$a\,\mathbf{v}_N, \mathbf{w}_N \quad \mathbf{f}, \mathbf{w}_N \;{}_{L_2(\Gamma)}, \quad \text{o} \quad \mathbf{w}_N \;\; \mathbf{S}_N,$$

n o C B/α o on o n y o n

o o c oy d d on on o n y nd on n y con
n zo d on o n y d n d o oc o nd c on
c o o on on **x** n y n on **y** **x**
on y on d n o o n do o nd y on
n x · a A **k → ∞** n on on co oc d o nd on
ncy n o Ny o od o nc od
no d on n o on n y o **k** o n c n n o n
o n o od o o c o y n

o y on no n y c n o n c n n c d
o c o o c c n cc y co n d o
co ncy c n y c n con o y on

n c on C^∞ con con o nd n n on
n y d d d y o n ncy o n T_1 nd T_2 n o n n d zon
nd do zon d c d n L n γ , $\pi \to$ π od c
z on o d n t , π o o T γ t T o
i , o c c n z y n n n v γ s , k
v s, k o con n nc

$$v\ s, k \quad k \quad k \gamma\ s \cdot a\ V\ s, k\ .$$

n c , π co d y o n , i , . . . , $_1$
nd $_2$ n y n o ood o n ncy o n t_1 nd t_2 nd $_3$ nd $_4$

[illegible]

For all $L, M \in \mathbb{N} \cup \{\ \}$, the function $V(s,k)$ admits a decomposition of the form:

$$V(s,k) \quad \left[\sum_{=0}^{L\,M} k^{-1\;3-2\;\;3-}\, b\;(s)\;\;(\,)\,(k^{1\;3} Z(s)) \right] \quad R_{L\,M}(s,k)\;,$$

for $s \in [\;,\ \pi]$, where the remainder term has its nth derivative bounded, for $n \in \mathbb{N} \cup \{\ \}$, by

$$|D^n R_{L\,M}(s,k)| \le C_{L\,M\,n}\;\; k^{\;\;+n\;3}\,,$$

where $\mu \;\; - \;\; \frac{2}{3}(L \;\;,\; M)$ and $C_{L,M\,n}$ is independent of k. The functions b and Z are C^∞ π-periodic functions. Z has simple zeros at t_1 and t_2

d c on n y n y o c n y y o c n y d d c d
y y n o y o d o con o n d n n] d]
L] Mo d n o n n y o c o
o o n o d o V d n

For all $n \in \mathbb{N} \cup \{\ \}$ there exist constants $C_n >$ independent of k and $s \in$, π , such that for all k sufficiently large,

$$|D^n V(s,k)| \leq C_n \begin{cases} , & n \quad , \ , \\ k^{-1}(k^{-1}\ 3 \quad |\omega(s)|)^{-n-2}, & n \geq \quad , \end{cases}$$

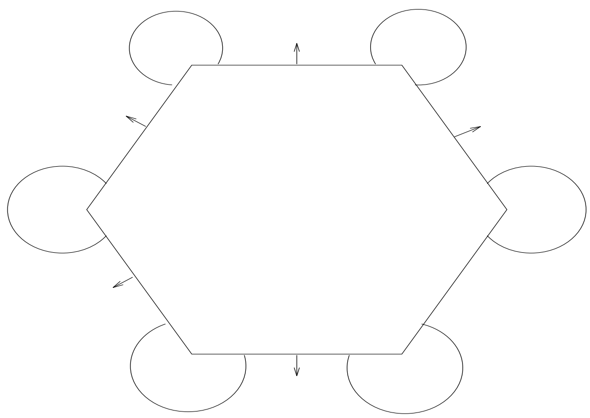
where $\omega(s)$ $(s - t_1)(t_2 - s)$. These estimates are uniform in $s \in$, π .

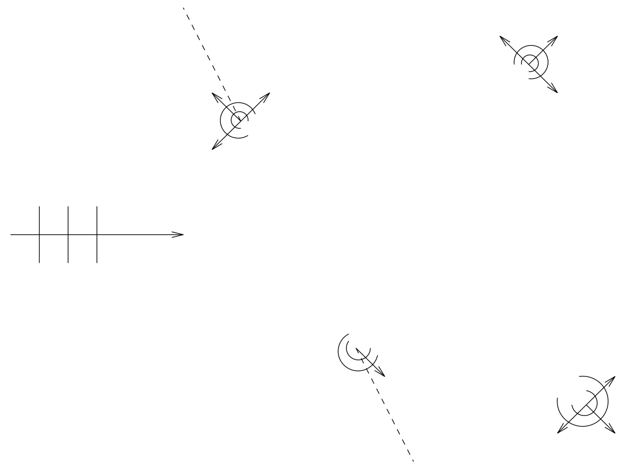
n o o o o n o o o n n o n c o o o s n n d zon nd o nd d y o t_1, t_2 nd nc $|\omega(s)|$

y c oo n **p** o o y n $k^{1\ 9}$ cc cy o od
k nc N c n o
B o n d c on n on n y o c n **s** n
o n t_1 n do on n n on o n n
n y o c $k \to \infty$

$$k^{-1\ 3} b_{0\ 0} \quad k|Z\ s|^{3}/ \quad \alpha_1\ k^{1\ 3}|Z\ s| \quad -\ \alpha_1\ k^{1\ 3}|Z\ s| \ .$$

nc α_1 n d n o co n con n o o c
o y c o on o c n c **k** nd on c $k^{1\ 3}$ d d y on n y
d c y n d o c od d n nc on d n co o
c on od o d nd c oo n o cco n nc o c n
nd do o nd y]
no n o n on o c n y con o y on od





n § n c n y o ncy ond y n od n o c on d on n n c on o c d o c ond o on n y o n o y con n **C** n no o on ond y n on od c c y d d o ncy c n o c on y y c o y n y nd cond on n o con n on c o yno o nd y n od ncy

no d dy n c n o **a** co n o nd on y con n **C** n c n $\mathbf{v}_N$ d n d y n od y

$$\mathbf{C} \leq \frac{\mathbf{B}}{\alpha}$$

B nd **α** con n y nd co y con n n d con n c o y d o no o **A** nd n o o on $\mathbf{L}^2$ nd d y c y c z o $\mathbf{v}, \mathbf{w} \in \mathbf{L}^2$ nd $\langle \cdot \rangle$ d no n o o no o nd d n o o on $\mathbf{L}^2$

$$|a(\mathbf{v},\mathbf{w})| = |(\mathbf{A}\mathbf{v},\mathbf{w})_{L^2(\Gamma)}| \leq \|\mathbf{A}\mathbf{v}\|_{L^2(\Gamma)} \|\mathbf{w}\|_{L^2(\Gamma)}$$
$$\leq \|\mathbf{A}\| \|\mathbf{v}\|_{L^2(\Gamma)} \|\mathbf{w}\|_{L^2(\Gamma)}.$$

$\|\mathbf{A}\|$ o o con n **B** n d n c o o o n **w** $\mathbf{A}\mathbf{v}$ n n y $\|\mathbf{A}\|$ o o con n **B** o c d o d y o cond o n d

$$\|\mathbf{A}\mathbf{v}\|_{L^2(\Gamma)} \|\mathbf{v}\|_{L^2(\Gamma)} \geq |(\mathbf{A}\mathbf{v},\mathbf{v})_{L^2(\Gamma)}| = |a(\mathbf{v},\mathbf{v})| \geq \alpha \|\mathbf{v}\|^2_{L^2(\Gamma)},$$

o

$$\|\mathbf{A}^{-1}\| \leq \alpha^{-1}.$$

o **B/α** o nd d o y **condition number** o o o **A**

$$\frac{\mathbf{B}}{\alpha} \geq \operatorname{cond} \mathbf{A} = \|\mathbf{A}\| \|\mathbf{A}^{-1}\|.$$

on on o dy n cond on n o **A** nd d nd nc on **k** c n o c o c on Ano on o o n n y on y **a** co c o no n o c c c o no o n y o no **a** co

$N \to \infty$ k

as $k \to \infty$, uniformly in $\eta >$.

$$\|S\| \leq \sqrt{\frac{a}{\pi k}}.$$

An o n c on o o n n n o
A on η c o n o

$$l_1 R_0^{-1}, l_2 k \le \eta \le u_1 R_0^{-1}, u_2 k,$$

o o o con n l_1 l_2 u_1 nd u_2 n o o con n $c >$ A^{-1} A^{-1} $\le$ c o $k >$ o c oo n

$$\eta \quad R_0^{-1} \quad k,$$

c l_1 l_2

conc n d on o y o n on o on o o z
on nd n c o on y n nd co oc on od n
ncy c A y o n on c on y c o d y
od c n d co on c on
$\mathbf{k} \to \infty$ n c do co on c o c c
n d c y n c on o on o d n c
d o on

n nd co oc on od d c d o on y o
n c n o c n o c o y n Ny o o c

no n S_n no on y o n nd P_n c n y ny n n

ncy o z o o d y **conventional** o nd y n
d n on o **hybrid** o on n o n nd c n n o
o n c y n nc n n n y d o on c
o o d o **N** y o o c n o z
n n on n d c n y d o n o on n
n y d c c o n o don o y d o

Bonn B [illegible] nd [illegible] co [illegible] on of con c [illegible] d c on co [illegible] n n [illegible] ncy co c [illegible] c n **SIAM J. Numer. Anal.**

B [illegible] nd n d [illegible] d c c A n o [illegible] d o z c c n n c n **Arch. Math.** [illegible]

B no [illegible] z n [illegible] A Mono A c [illegible] c d o o nc n [illegible] d co [illegible] on [illegible] o c n o [illegible] y [illegible] ncy co [illegible] c **Phil. Trans. R. Soc. Lond. A.** [illegible]

B no [illegible] nd z n [illegible] A [illegible] An **O** n on c [illegible] o d n on [illegible] c c n o **J. Comp. Appl. Math.** [illegible]

B no [illegible] z n [illegible] A c [illegible] n **O** o on of [illegible] c n o **IEEE Trans. Magn.** [illegible]

B no [illegible] nd ny n y L [illegible] c c n n d n on n cc d o d o **Proc. R. Soc. Lond.** [illegible]

B no [illegible] co [illegible] n c on

B A nd [illegible] n co c n y o n [illegible] on nd o n y **SIAM J. Sci. Comput.** [illegible]

B [illegible] d o [illegible] y o c [illegible] o n o [illegible] d c on y o con con o n n **Trudy Mat. Inst. Steklov.** [illegible] A [illegible] d n [illegible] y n o [illegible] y o c n o [illegible] d c on y con od **Soviet Physics Doklady** [illegible]

B [illegible] d [illegible] y o c [illegible] o of c c c c o o o [illegible] o c od n o o n n **Izv. Akad. Nauk SSSR Ser. Mat.** [illegible] n n on d **Math. USSR–Izv.** [illegible]

[illegible] nd d N [illegible] L n don nd L ndn M [illegible] ond on n [illegible] o co [illegible] n d o n o nd y n o o n co c c n c N on n o M [illegible] c c nc n N [illegible] c N on n

[illegible] nd d N [illegible] L n don nd L ndn M [illegible] ond on n [illegible] o co n d o n n o o n co c nd o nd y n d c on n on

[illegible] nd d N L n don nd L A [illegible] n o nd y n [illegible] od o n co c c n o **Phil. Trans. R. Soc. Lond. A.** [illegible] nd d N nd L n don A n o nd y n od o ncy c n y co o y on **SIAM J. Numer. Anal.**

nd d N nd Mon n c o nd n n c c n **SIAM J. Math. Anal.**

z n d on c on d d o y o o on d ond **C. R. Acad. Sc. Paris**

on M nd B o c co n n c o n c n on **Computer Modeling in Engineering and Sciences**

o on nd d **Integral Equation Methods in Scattering Theory** y N Yo

nd o n o o od nd c o oc d c z on o o z on **J. Comput. Phys.**

d nd A o od o M on nc **Phil. Trans. R. Soc. Lond. A**

o n z nd y A y d n c y o c o nd y n od o ncy co c c n **Numer. Math.**

n on o on o c o n c nd co c c n o co nc o c n on nd ncy y o c n on n y o M nn o

An y o c n on o ncy c n o o d n on c o M nc n M n d n N n c n L z

B od M nd N M d o n c n o o nd y o on C^1 do n **Acta Math.**

L d c c n nd d A n d c n n c n **Math. Z.**

M n M nd L n don An **hp** B M o ncy c n y con o y on oc d n o A n y o d n

M o B y o M d N c n nd co c d c o

] y A B d o o c on o d nd o
J. Sov. Math.

d o M dd ncy c n y con o c **Duke Math. J.**