const nt ort ity e note th t independent y on the ue of the ort ity m the considered cont ct ode e hi its ery strong c ustering th t is re ected in the ound on the corre tion functions t ny o ent of ti e t Note th t this e ect on the e e of the co puter si u tion s disco ered re dy in nd no it h s the rigorous the tic for u tion nd c ri c tion A direct consequence of the co petition in the ode is the suppression of such c ustering N e y ssu ing the strong enough co petition nd the ig intrinsic ort ity m e pro e the su -Poissoni n ound for the so ution to the o ent equ tions pro ided such ound s true for the initi st te Moreo er e c rify speci c in uences of the const nt nd the density dependent ort ity intensities sep r te y More precise y the ig enough intrinsic ort ity m gi es unifor in ti e ound for e ch corre tion function nd the strong co petition resu ts ensure the regu r sp ti distri ution of the typic con gur tion for ny o ent of ti e th t is re ected in the su -Poissoni n ound oint in uence of the intrinsic ort ity nd the co petition e ds to the e istence of the unique in ri nt e sure for our ode hich is just Dir c e sure concen-tr ted on the e pty con gur tion The tter e ns th t the corresponding stoch stic e o ution of the popu tion is sy ptotic y e h usting

The e sure μ , $K^{□}$ is c ed the corre tion e sure of
As sho n in for □ $\mathbf{M}^{1}_{\mathrm{fm}}$ nd ny G □ L^{1} $_{0}$ μ the series is , s
so ute y con ergent Further ore KG □ L $_{(4G}$

here

$$D_x^+ F \quad F \quad x - F$$

nd $\varkappa^+$ is so e positi e const nt

The e istence of the process ssoci ted ith L_+ c n e sho n using the s e technique s in Let $_t$ e the corresponding e o ution of e sures in ti e on $\mathbf{M}^1_{\mathrm{fm}}$ By $k_t^{(n)}$ $n \geq$ e denote the dyn ics of the corresponding n th order corre tion functions pro ided they e ist Note th t e ch of such functions descri es the density of the syste t the o ent t

Then using for ny continuous on $\mathbb{R}^d$ ith ounded support e o t in

$$\frac{d}{dt}\int_{\mathbb{R}^d} x \; k_t^{(1)} \; x \; dx \quad \frac{d}{dt}\int_\Gamma \quad d \; _t \quad \int_\Gamma L_+ \quad d \; _t$$

$$\varkappa^+ \int_\Gamma a^+ \quad d \; _t \quad \varkappa^+ \int_{\mathbb{R}^d} a^+ \quad x \; k_t^{(1)} \; x \; dx$$

$$\varkappa^+ \int_{\mathbb{R}^d} x \quad a^+ \; k_t^{(1)} \quad x \; dx$$

here denotes the c ssic con o ution on $\mathbb{R}^d$ Hence $k_t^{(1)}$ gro s e ponenti y in t In p rticu r for the tr ns tion in ri nt c se one h s $k_0^{(1)}$ x $\equiv k_0^{(1)}$ nd s resu t

$$k_t^{(1)} \quad e^{\varkappa^+ t} k_0^{(1)}$$

One of the possi i ities to pre ent the density gro th of the syste is to inc ude the de th ech nis The si p est one is descri ed y the independent de th r te ort ity m This e ns th t ny e e ent of popu tion h s n independent e ponenti y distri uted ith p r eter m r ndo ife ti e The independent de th together ith the independent cre tion of ne p rtic es y re dy e isting ones de scri e the so c ed *contact model* in the continuu see e g The pre gener tor of such ode is gi en y the fo o ing e pression

$$L_{\mathrm{CM}} F \quad m \sum_{x \in \gamma} D_x^- F \quad L_+ F$$

$$m \sum_{x \in \gamma} D_x^- F \quad \varkappa^+ \sum_{y \in \gamma} \int_{\mathbb{R}^d} a^+ \; x - y \; D_x^+ F \quad dx$$

here

$$D_x^- F \quad F \quad \setminus x - F$$

The M r o process ssoci ted ith the gener tor L_{CM} s constructed in This construction s gener ized in for ore gener c sses of functions a^+ Let us note th t the cont ct ode in the continuu y e used in the epide io ogy to ode the infection spre ding process The ues of this process represent the st tes of the infected popu tion This is n og of the cont ct process on ttice Of course such interpret tion is not in the sp ti eco ogy concept On the other h nd cont ct process is sp ti r nching process ith gi en ort ity r te

The dyn ics of corre tion functions in the cont ct ode s considered in N e y t ing m for correctness e h e for ny $n \geq$ t the corre tion function of n th order h s the fo o ing for

$$k_t^{(n)}(x_1,\dots,x_n) = e^{n(\varkappa^+-1)t}\left[\bigotimes_{i=1}^{n} e^{tL^i_{a^+}}\right] k_0^{(n)}(x_1,\dots,x_n) + \varkappa^+\int_0^t e^{n(\varkappa^+-1)(t-s)}\left[\bigotimes_{i=1}^{n} e^{(t-s)L^i_{a^+}}\right] \times \sum_{i=1}^{n} k_s^{(n-1)}(x_1,\dots,\check{x}_i,\dots,x_n)\sum_{j:\,j\neq i} a^+(x_i-x_j)\,ds$$

here

$$L^i_{a^+}k^{(n)}(x_1,\dots,x_n) = \varkappa^+\int_{\mathbb{R}^d} a^+(x_i-y)\left[k^{(n)}(x_1,\dots,x_{i-1},y,x_{i+1},\dots,x_n) - k^{(n)}(x_1,\dots,x_n)\right]dy$$

nd the sy o $\check{x}_i$ e ns th t the i th coordin te is o itted Note th t $L^i_{a^+}$ is M r o gener tor nd the corresponding se igroup in L^∞ sp [illegible] preser esr e consider $t \geq$ One c n pro e y induction th t for ny $\{x_1, \ldots, x_n\} \square B$ $n \geq$

$$k_t^{(n)} \; x_1, \ldots, x_n \;\; \geq \;\; {}^{n} e^{n(\varkappa^+ - 1)t} n^{!}$$

Indeed for n this st te ent h s een pro ed Suppose th t ho ds for $n -$ Then y one h s

$$k_t^{(n)} \; x_1, \ldots, x_n \;\; \geq \varkappa^+ \int_0^t e^{n(\varkappa^+ - 1)(t-s)} n \;\; {}^{(n-1)} e^{(n-1)(\varkappa^+ - 1)s} \; n - \;\; ! \; n - \qquad ds$$

$$\geq \;\; {}^{n} n! e^{n(\varkappa^+ - 1)t} \int_0^t e^{-(\varkappa^+ - 1)s} ds \geq \;\; {}^{n} e^{n(\varkappa^+ - 1)t} n!$$

As it s entioned efore the ter ound sho s the c ustering in the cont ct ode A pre ious consider tion y e e tended for the c se $m \square$ e shou d on y rep ce y m in the pre ious c cu tions

As conc usion e h e the presence of ort ity m $\varkappa^+$ in the free gro th ode pre ents the gro th of density i e the corre tion functions of orders dec y in ti e But it doesn t in uence on the c ustering in the syste One of the possi i ities to pre ent such c ustering is to consider the so c ed density dependent de th r te N e y et us consider the fo o ing pre gener tor

$$LF \qquad \sum_{x \square \gamma}^{2}$$

Proof. It is not di cu t to sho th t L_0 is dense y de ned nd c osed oper tor in $\mathbf{L}_C$.

Let $\frac{\pi}{2}$ e r itr ry nd ed C e r th t for ζ Sect $\frac{\pi}{2}$

$$m| \, | \, \varkappa^{-} E^{a^{-}} , \quad \zeta$$

Therefore for ny ζ Sect $\frac{\pi}{2}$ the in erse oper tor $L_0 - \zeta$ $^{-1}$ the ction of hich is gi en y

$$L_0 - \zeta \quad ^{-1} G \, , \qquad - \frac{}{m| \, | \quad \varkappa^{-} E^{a^{-}} , \quad \zeta} G \, ,$$

is e de ned on the ho e sp ce $\mathbf{L}_C$ Moreo er it is ounded oper tor in this sp ce nd

$$\| L_0 - \zeta \quad ^{-1} \| \leq \begin{cases} \frac{1}{|\zeta|} & \text{if } \mathrm{Re}\,\zeta \geq \\ \frac{M}{|\zeta|} & \text{if } \mathrm{Re}\,\zeta \end{cases}$$

here the const nt M does not depend on ζ

The c se $\mathrm{Re}\,\zeta \geq$ is direct consequence of nd inequ ity

$$m| \, | \quad \varkappa^{-} E^{a^{-}} , \quad \mathrm{Re}\,\zeta \geq \mathrm{Re}\ \zeta \geq$$

e pro e no the ound in the c se $\mathrm{Re}\,\zeta$ sing e h e

$$\| L_0 - \zeta \quad ^{-1} G \|_C \quad \left\| \frac{}{m| \cdot | \quad \varkappa^{-} E^{a^{-}} \cdot \quad \zeta} G \cdot \right\|_C$$

$$\frac{}{|\zeta|} \left\| \frac{|\zeta|}{m| \cdot | \quad \varkappa^{-} E^{a^{-}} \cdot \quad \zeta} G \cdot \right\|_C$$

Since ζ Sect $\frac{\pi}{2}$

$$|\mathrm{I} \quad \zeta| \geq |\zeta| \sin - \quad |\zeta| \cos$$

Hence

$$\frac{|\zeta|}{m| \, | \quad \varkappa^{-} E^{a^{-}} , \quad \zeta} \leq \frac{|\zeta|}{|\mathrm{I} \quad \zeta|} \leq \frac{}{\cos} \quad M$$

nd is fu ed

The rest of the st te ent of the e fo o s direct y fro the theore of Hi e Yosid see e g ∎

e de ne no

$$L_1 G \, , \quad \varkappa^{-} \sum_{x \in \eta} \sum_{y \in \eta \setminus x} a^{-} \ x - y \ G \, , \ \setminus y \qquad G \in D \ L_1 \quad D \ L_0$$

The e e o i p ies th t the oper tor L_1 is e de ned

Lemma *For any the*

Proof. By odu us property

$$\|L_1 G\|_C \quad \int_{\Gamma_0} |L_1 G \quad | C^{|\eta|} \quad d$$

c n e esti ted y

$$\varkappa^- \int_{\Gamma_0} \sum_{x \in \eta} \sum_{y \in \eta \setminus x} a^- \; x - y \; |G \quad \setminus y \; | C^{|\eta|} \quad d$$

By Min os e is equ to

$$\varkappa^- \int_{\Gamma_0} \int_{\mathbb{R}^d} \sum_{x \in \eta} a^- \; x - y \; |G \quad | C^{|\eta|+1} dy \quad d$$

$$\varkappa^- \int_{\Gamma_0} C| \; \|G \quad | C^{|\eta|} \quad d \quad \leq \frac{\varkappa^-}{m} C \|L_0 G\|_C$$

Therefore ho ds ith

$$a \quad \frac{\varkappa^- C}{m}$$

C e r th t t ing

$$C_0 \quad \frac{m}{\varkappa^-}$$

e o t in th t a for C C_0 □

e set no

$$L_2 G \quad \quad L_{2,\varkappa^+} G \quad \quad \varkappa^+ \int_{\mathbb{R}^d} \sum_{y \in \eta} a^+ \; x - y \; G \quad \setminus y \quad x \; dx$$

$$G \in D \; L_2 \quad D \; L_0$$

The oper tor de ned s

$$NG \quad | \, |G \qquad G \in D \; L_0$$

is c ed the nu er oper tor

Remark *We proved, in particular, that for* $G \in D\; L_0 \quad D\; L_1 \quad D\; L_2$

$$\|L_1 G\|_C \leq \varkappa^- C \|NG\|_C$$
$$\|L_2 G\|_C \leq \varkappa^+ \|NG\|_C.$$

Fin y e consider the st p rt of the oper tor L.

$$L_3 G \quad \varkappa^+ \int_{\mathbb{R}^d} \sum_{y \in \eta} a^+ \; x - y \; G \; \cup x \; dx \quad D\; L_3 \quad D\; L_0.$$

Lemma *For any and any* $\varkappa^+$ *,* C *such that*

$$\varkappa^+ E^{a^+} \qquad C \;\; \varkappa^- E^{a^-} \qquad m| \; |$$

the following estimate holds

$$\|L_3 G\|_C \leq a \|L_0 G\|_C \qquad G \in D\; L_3$$

with $a \quad a \; \varkappa^+ \; C$.

Proof. sing the s e tric s s in the t o pre ious e s e h e

$$\|L_3 G\|_C \qquad \int_{\Gamma_0} |L_3 G \;\; | C^{|\eta|} \;\; d$$
$$\leq \varkappa^+ \int_{\Gamma_0} \int_{\mathbb{R}^d} \sum_{y \in \eta} a^+ \; x - y \; |G \; \cup x \; | C^{|\eta|} dx \quad d \; .$$

By Min os e is equ to

$$\frac{\varkappa^+}{C} \int_{\Gamma_0} E^{a^+} \;\; |G \;\; | C^{|\eta|} \;\; d \; .$$

The ssertion of the e is no tri i □

Theorem *Assume that the functions* $a^- \; a^+$ *and the constants* $\varkappa^- \; \varkappa^+$ *,* m *and* C *satisfy*

$$C \varkappa^- a^- \geq \varkappa^+ a^+,$$
$$m \qquad \varkappa^- C \quad \varkappa^+ .$$

Then, the operator L *is a generator of a holomorphic semigroup* $U_t \;\; t \geq$ *in* $\mathbf{L}_C$.

Proof. The st te ent of the theore fo o s direct y fro Re r Le nd the theore out the pertur tion of ho o orphic se igroup see e g For the re der s con enience e o e gi e its for u tion.

For any $T \in \mathbf{H}$ *and for any* *there exists positive constants* , *such that if the operator* A *satisfies*

$$\|Au\| \leq a \|Tu\| \quad b \|u\| \qquad u \in D\; T \quad \in D\; A$$

with a , b , *then* T

Let us denote for η □ □

$$Sk\ \eta \qquad - \frac{\varkappa^-}{m|\ |\ \ \varkappa^- E^{a^-}\ \eta} \sum_{x\square\eta} \int_{\mathbb{R}^d} k\ \ y\ \square\eta\ \ \ a^-\ \ x-y\ \ dy$$
$$\frac{\varkappa^+}{m|\ |\ \ \varkappa^- E^{a^-}\ \eta} \sum_{x\square\eta} \sum_{y\square\eta\setminus x} a^+\ \ x-y\ \ k\ \eta\ \ \setminus x$$
$$\frac{\varkappa^+}{m|\ |\ \ \varkappa^- E^{a^-}\ \eta} \int_{\mathbb{R}^d} \sum_{y\square\eta} a^+\ \ x-y\ \ k\ \ \eta\ \ \setminus y\ \ \square x\ \ dx$$

nd

$$Sk\ \ \square\ \ \ \ \eta$$

Let

$$\square k\square_C \qquad \operatorname{ess\,sup}_{\eta\square\Gamma_0} \frac{|k\ \eta\ \ |}{C^{|\eta|}}$$

then

$$\square Sk\square_C$$
$$\leq \quad \square k\square_C \operatorname{ess\,sup}_{\eta\square\Gamma_0\setminus\{\square\}} \frac{\varkappa^- C}{m|\ |\ \ \varkappa^- E^{a^-}\ \eta} \sum_{x\square\eta} \int_{\mathbb{R}^d} a^-\ \ x-y\ \ dy$$
$$\frac{\square k\square_C}{C} \operatorname{ess\,sup}_{\eta\square\Gamma_0\setminus\{\square\}} \frac{\varkappa^+}{m|\ |\ \ \varkappa^- E^{a^-}\ \eta} \sum_{x\square\eta} \sum_{y\square\eta\setminus x} a^+\ \ x-y$$
$$\square k\square_C \operatorname{ess\,sup}_{\eta\square\Gamma_0\setminus\{\square\}} \frac{\varkappa^+}{m|\ |\ \ \varkappa^- E^{a^-}\ \eta} \int_{\mathbb{R}^d} \sum_{y\square\eta} a^+\ \ x-y\ \ dx$$
$$\leq \quad \square k\square_C \frac{C\varkappa^-}{m} \quad \square k\square_C \frac{\varkappa^+}{m} \quad \square k\square_C \frac{}{C} \quad \square k\square_C$$

Then

$$\hat{L}G \quad \begin{cases} \varkappa^{+} \int_{\mathbb{R}^d} \sum_{y \in \eta} a^{+}\, x - y\; G^{(n)} \quad x\, dx & | \;| \quad n - \\ -\varkappa^{-} \sum_{x \in \eta} \sum_{y \in \eta \setminus x} a^{-}\, x - y\; G^{(n)} \quad \setminus y & | \;| \quad n \\ L_{diag} G^{(n)} & | \;| \quad n \\ & \text{other ise} \end{cases}$$

Note th t sgn G $\equiv$ if $| \;| \; n$

Therefore for r itr ry $n \geq$

$$\geq I_n \int_{\Gamma_0} \text{sgn}\; G \quad \cdot \quad L - b \qquad G \quad \cdot \qquad b \qquad n)$$

Since E^{a^-} , for | | ≤ e get

$$\geq \sum_{n=1} I_n - m \sum_{n=1} n \frac{t^n C^n}{n!} | |^n - \varkappa^- \sum_{n=1} \frac{t^n C^n}{n!} \int_{\Lambda^n} E^{a^-} x^{(n)} dx^{(n)}$$

$$\varkappa^+ \sum_{n=1} \frac{t^n C^n}{n!} n | |^{n-1} \int_\Lambda \int_\Lambda a^+ x - y \, dxdy - b \sum_{n=1} \frac{t^n C^n}{n!} | |^n$$

$$- mtC | | e^{Ct|\Lambda|} - \varkappa^- \int_{\Gamma_\Lambda} E^{a^-} , d_{Ct} , \varkappa^+ Cte^{Ct|\Lambda|} \int_\Lambda \int_\Lambda a^+ x - y \, dxdy$$

$$- b \left[e^{Ct|\Lambda|} - \right]$$

$$- mtC | | e^{Ct|\Lambda|} - \varkappa^- C^2 t^2 \int_{\Gamma_\Lambda} \int_\Lambda \int_\Lambda a^- x - y \, dxdyd_{Ct} ,$$

$$\varkappa^+ Cte^{Ct|\Lambda|} \int_\Lambda \int_\Lambda a^+ x - y \, dxdy - b \left[e^{Ct|\Lambda|} - \right] .$$

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