University of Reading

# Modelling of Hot Water Flooding

as an

## Enhanced Oil Recovery Method

by

Zeinab Zargar

August 2013

Department of Mathematics

Submitted to the Department of Mathematics, University of Reading,
in Partial ful lment of the requirements for the Degree of Master of Science in
Mathematics of Scienti c and Industrial Computation

# Acknowledgment

# Declaration

I con rm that this is my own work and the use of all materials from other sources has been properly and fully acknowledged.

Signed .....

# Abstract

# Contents

# 1 Introduction

A hydrocarbon reservoir is an underground volume comprised of porous rock containing a mixture of water and hydrocarbon uids in the form of oil and gas, occupying the void space of the pores in the rock. Oils can be divided into two categories, light oils and heavy oils. Light oils have a low viscosity while heavy oils have a high viscosity. The viscosity of a uid is a measure of how easily that uid will ow, for instance, water has a very low viscosity while honey has a high viscosity.

When oil recovery is high due to high natural reservoir pressure. The rate of natural oil production will diminish with time, but there are some oil recovery methods to improve the production rate. Oil recovery processes involve the injection of uid or a combination of uid and chemicals into the oil reservoir via injection wells to force as much oil as possible towards and, hence, out of the production wells. Light oils are extracted under primary and secondary recovery methods which involve allowing the uid to ow out under the natural pressure of its surrounding. These methods cannot be applied to the extraction of heavy oils, whose viscosity is far too high for such methods to be e ective; their viscosity needs to be reduced. This is achieved by various thermal stimulation techniques like hot water ooding, steam injection, in-situ combustion and so far which raise the temperature of the oil, e ectively reducing its viscosity. The approach which is under consideration here is hot water injection modeling. It is necessary to model and simulate this process in order to provide information about production and the future of the reservoir to get the best recovery.

All thermal recovery processes tend to raise the temperature of the crude in a reservoir to reduce the reservoir ow resistance by reducing the viscosity of the crude [12]. It is desirable to heat the reservoir e ciently, but inevitably some of the heat in the reservoir is lost through produced uids, and some is lost to the adjacent overburden and underburden formations. The heat loss to the adjacent formations is controlled by conduction (heat transfer) which it can be readily estimated.

In hot water ooding, as can be seen in gure 1.1 many reservoir equivalent volumes of hot water are injected into a number of wells in order to reduce the viscosity and subsequently displace the oil in place more easily towards oil production wells. Hot water injection may be preferred in shallow reservoirs containing oils in the viscosity range of 100-1000 cp [4].

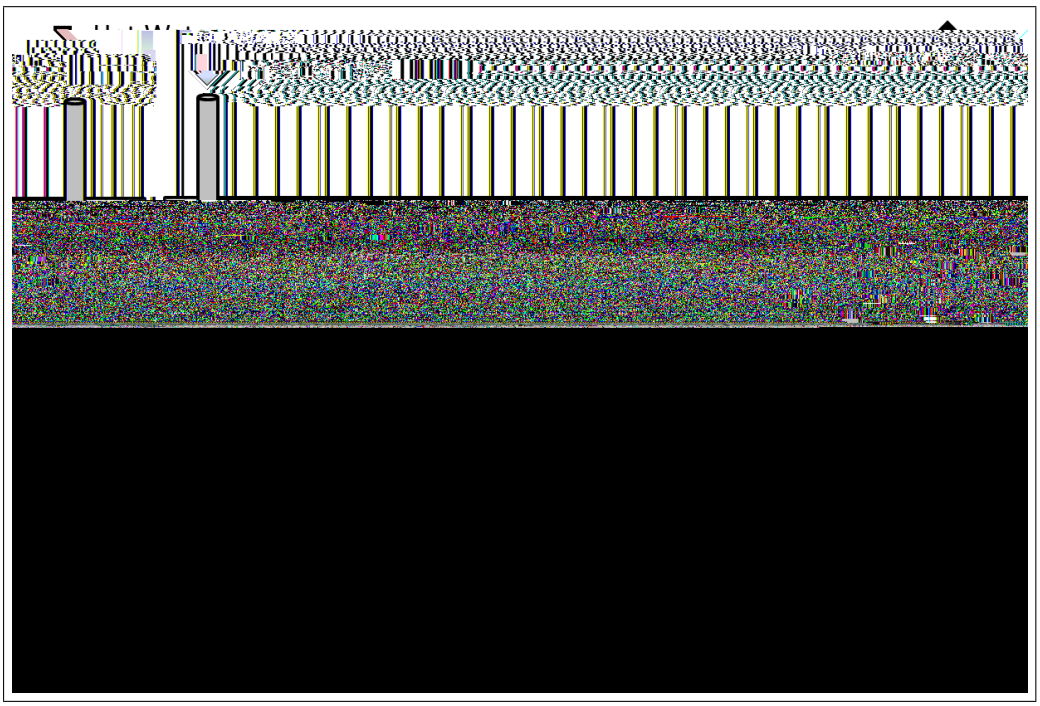


Figure 1.1: Schematic diagram of hot water injection process

The mathematical model representing the physical process of hot water injection requires rock and uid properties in order to describe the uid ow and heat transfer with a set of partial di erential equations and algebraic equations, which are derived from physical principals. This set of equations is derived from four main principles: Conservation of mass of phases (water and oil); Darcy's Law for volumetric ow rates which describes how the uid phases ow through the reservoir; volume balance equation, a condition which states that the uid lls the rock pore volume; conservation of energy of phases. Since the resulting equations are too complex for more realistic models to be solved using analytic techniques, here is focused on numerical techniques.

In this dissertation, two di erent models are applied and analyzed for the hot water injection process. Chapter 2 contains the problem de nition and characteristics of the model. Introducing some necessary concepts about rock and uid properties, and required equations. Initial and boundary conditions and heat loss in our model are also included in this part. The rst model is introduced in chapter 3, where in order to nd the saturation distribution the Buckley-Leverett equation is used. The nonlinear mass balance equation is solved by a fully implicit central technique by using the results of oil and water saturations from the Buckley-Levertt equation. Subsequently, the saturation and pressure results are applied to a nonlinear energy equation discretized by a fully implicit method. Finally, the mass balance equation

# 2 Characteristics of the Model

In this project, we have tried to model the hot water flooding process in a reservoir which is initially saturated with oil and water. The reservoir is considered to be one-dimensional between an injection and production wells. A schematic diagram of the model is given in figure 2.1. Hot water is injected with a constant rate and temperature into the porous media which is filled with cold and heavy oil. In such a system fluid flow, heat transfer and heat losses are modeled in order to give a better understanding of the process and its effect on oil recovery.

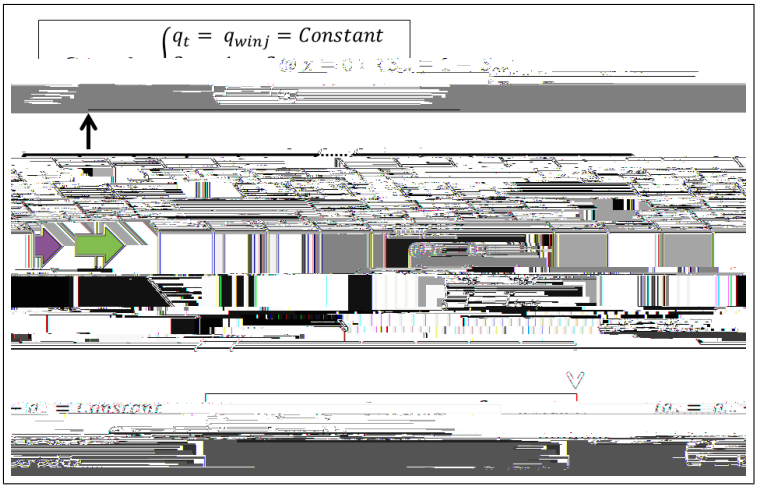



Figure 2.1: Schematic diagram of the problem

## 2.1 Assumptions

The following assumptions are made to model the process;

1. In all reservoir processes, every point within the reservoir is in thermodynamics equilibrium.
2. The injected fluid reaches thermal equilibrium instantaneously with the reservoir fluids and sand, meaning that all phases and rock in the same location have the same temperature.

$$' = \frac{\text{Pore Volume } (V_p)}{\text{Bulk Volume } (V_b)} \tag{2.4}$$

### 2.2.3 Saturation

The pore volume space is not always lled with a single uid. Saturation of each uid (phase) is de ned as the ratio of its volume over the total pore volume occupied by all phases [1],

$$S_i = \frac{\text{Phase Volume}(V_i)}{\text{Pore Volume}(V_p)} \tag{2.5}$$

By de nition, the saturations are all non-negative, and sum to one.

### 2.2.4 Permeabilities

One of the main properties of porous rock is its capability to allow uid ow through its connected pores which is known as permeability. There are two de nitions of permeability in the oil industry; absolute and relative permeabilities. Under the condition of single phase ow, this capability is named absolute permeability. But when the porous media is lled by more than one phase, due to various ways the phases can occupy the pore volume, the phases adversely a ect the ow of each other in a complicated manner [1]. This e ect is described using phase relative permeabilities, $K_{ro}$ and $K_{rw}$. The dependence of the relative permeabilities on the rock and uid properties is very complicated [2]; the $K_{ro}$ and $K_{rw}$ considered here are non-negative functions of the saturation, Sde nition, tls

$S_{wi}$: Irreducible water saturation

$S_{wc}$: Connate water saturation

$S_{orw}$: Residual water saturation (water-oil system )

$S_w$: Water saturation

$S_{wn}$: Normalized water saturation

Figure 2.2 shows the results of water and oil relative permeabilities versus water saturation in the system by applying the Corey correlation.In describing two-phase ow mathematically, it is always the relative permeability ratio, $\frac{K_{ro}}{K_{rw}}$, versus water saturations (for oil and water system) that enters the equations.

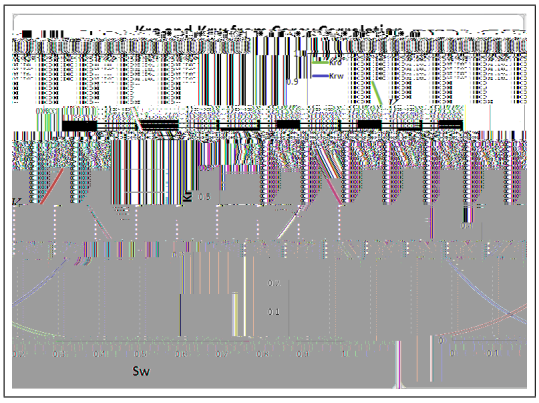



Figure 2.2: Oil and water relative permeabilities using Corey correlation

### 2.2.5 Hydrocarbon Viscosity

Phase viscosity represents the resistance of a phase to ow under the in uence of a pressure gradient. The most obvious e ect of thermal recovery on a reservoir uids is the reduction of oil viscosity. In gure 2.3 two points are evident. First, the rate of viscosity improvement is greatest as the initial temperature increases. Little viscosity bene t is gained after reaching a certain temperature. Second, greater viscosity reductions are experienced in the more viscous low API gravity crudes (API is a degree of measurement for oil density) than in higher API gravity crudes. Heating from $100^oF$ to $200^oF$ reduces the viscosity, 98% for $10^o$API crudes but only 73% for $30^o$API oils. These observations show that the greatest viscosity reduction occurs with the more viscous oils at the initial temperature increases [4].

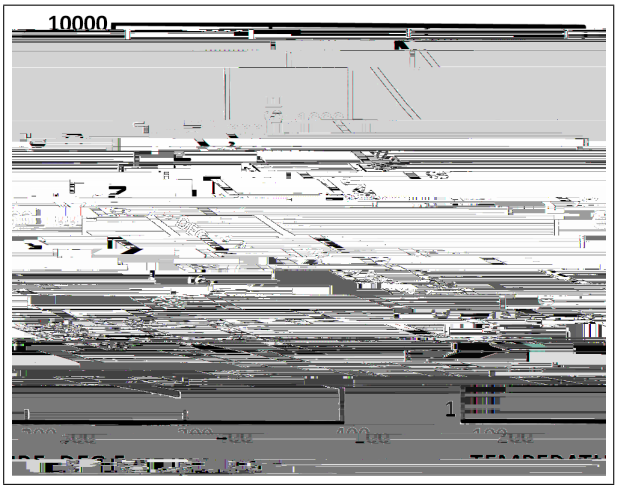



Figure 2.3: E ect of temperature on viscosity

Viscosity is a function of temperature and pressure, but water and oil viscosities are stronger functions of temperature in a thermal process rather than pressure. Since a thermal oil recovery method is modeled in this project, the e ect of pressure is neglected [5].

$$_{o} = 2{:}626 \quad 10^{8} \ (T \quad 459{:}59)^{\ 2{:}91} \tag{2.9}$$

$$_{w} = \frac{2{:}185}{0{:}04012\,(T \quad 459{:}59) + 5{:}154 \quad 10^{\ 6}\,(T \quad 459{:}59)^{2} \quad 1} \tag{2.10}$$

### 2.2.6 Phase Mass Density

Phase density is de ned as mass per unit volume for each phase. Water and oil densities in this context are considered to be a function of temperature and pressure. In the absence of experimental data, empirical relations are used to express densities of oil and water as functions of both temperature and pressure [5]:

$$_{w} = 63 \exp \ 17{:}253 \quad 10^{\ 5}\,(T \quad 459{:}59) \ \exp \ 4 \quad 10^{\ 6}\,(P \quad 1000) \tag{2.11}$$

$$_{o} = 59 \exp \quad 7{:}5885 \quad 10^{\ 5}\,(T \quad 459{:}59) \ \exp \ 1 \quad 1^{\ 5}\,(P \quad 1000) \tag{2.12}$$

Figure 2.4 shows the e ect of temperature and pressure increase on the densities of both phases.

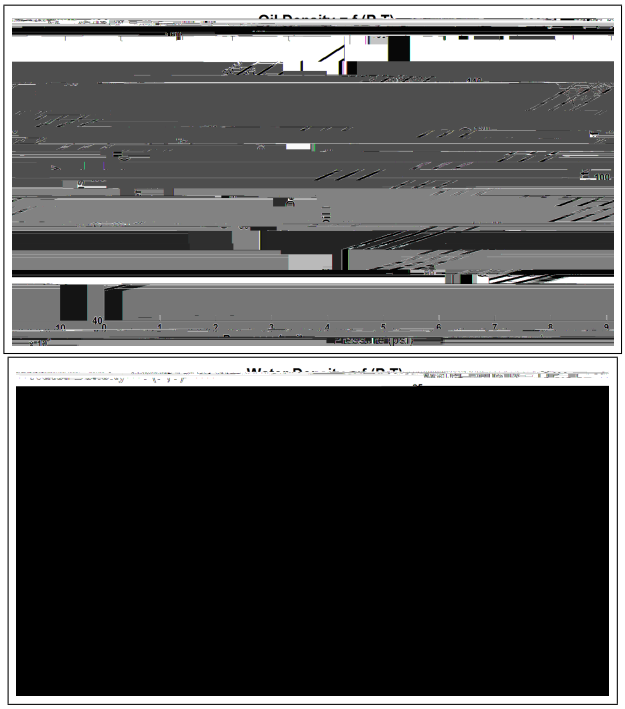


Figure 2.4: E ect of temperature and pressure on oil and water densities

## 2.3 Introducing Model Equations

In the two hot water models presented in this dissertation, four equations are required; the Buckley-Leverett equation, mass balance equations for water and oil, and an energy Balance equation.

### 2.3.1 Buckley-Leverett

The Buckley-Leverett (BL)equation is used in oil recovery in order to nd the saturation distribution in 1D reservoir. In the BL mechanism oil is displaced by water from a rock in a similar as uid is displaced from a cylinder by a leaky piston. In order to have better understanding of the Buckley-Leverett equation, it is rst necessary to introduce the fractional ow equation.

#### 2.3.1.1 Derivation of Fractional Flow for the Model

When oil is displaced by water in the system, from Darcy's equation we have

$$q_w = 1{:}127\, K_{abs} K_{rw}$$

$$q_o = \quad 1{:}127\ K_{abs}\frac{K_{ro}}{o}A_x \quad \frac{@P}{@x} \tag{2.14}$$

By adding the two equations

$$q_w + q_o = \quad 1{:}127\ K_{abs}A_x \quad \frac{K_{rw}}{w} + \frac{K_{ro}}{o} \quad \frac{@P}{@x} \tag{2.15}$$

Substituting for

$$q = q_w + q_o \tag{2.16}$$

and

$$f_w = \frac{q_w}{q} \tag{2.17}$$

and solving for the fraction of water owing, we obtain

$$f_w = \frac{1}{1 + \frac{K_{ro}}{o} : \frac{w}{K_{rw}}} \tag{2.18}$$

since

$$K_r\ (S_w)\ ) \quad f_w(S_w) \tag{2.19}$$

Now, the Buckley-Leveret equation is derived for a 1D sample based on mass conservation and some assumptions [6], namely ow is linear and steady state, the uid is incompressible, capillary pressure ($P_c$) is just a function of the saturation and pressure gradient for two phases is equal ($\frac{dP_c}{dS} = 0$), where $P_c = P_o \quad P_w$.

By applying mass balance of water around a control volume (see gure 2.5) of length 4 x we get the following system for a time period of 4 t :

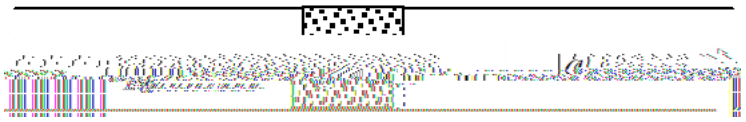


Figure 2.5: Mass Balance Element for Fractional Flow Equation

The material balance may be written:

$$\left[\ (\ _w q_w)_x \quad (\ _w q_w)_{x+4\,x}\ \right] 4\,t = A4\,x' \left[\ (\ _w S_w)^{t+\ t} \quad (\ _w S_w)^t\ \right] \tag{2.20}$$

which, when x ! 0 and t ! 0 , reduces to the continuity equation:

$$\frac{@}{@x}(\ _w q_w) = A' \ \frac{@}{@t}(\ _w S_w) \tag{2.21}$$

by assuming an incompressible uid $_w$ = constant and we have that $q_w = f_w q$, Therefore

$$\frac{@f_w}{@x} = \frac{A'}{q}\frac{@S_w}{@t} \tag{2.22}$$

Since $f_w(S_w)$, equation 2.22 may be rewritten as [6]

$$\frac{@f_w}{@S_w}\frac{@S_w}{@x} = \frac{A'}{q}\frac{@S_w}{@t} \tag{2.23}$$

where fractional water ow is de ned as

$$\begin{cases} f_s(S_{cv}) = \dfrac{1}{1+\frac{K_{ro}}{K_w}:\frac{w}{o}} = \dfrac{1}{1+\frac{(K_{ro})_{max}}{K_{rw}}:\left(\frac{1-S_n}{S_n}\right)^3:\frac{w}{o}} \\ S_n = \dfrac{S_w - S_{wc}}{1 - S_{wc} - S_{or}} \qquad S_{wc} \end{cases}$$

### 2.3.2 Mass Balance Equation

The main ow equation in reservoir engineering can simply be derived by applying material balance to a control volume, as shown in gure 2.7. Mass accumulation inside a control volume is the di erence between input and generated mass and output and consumed mass as below:

$$(\dot{m}_i \quad \dot{m}_o) \quad (\dot{m}_{cons} \quad \dot{m}_{gen}) = \frac{@M}{@t} \quad ; \quad \dot{m} = \text{Mass Flux} = \quad : q \tag{2.26}$$

whereq is ow rate , is density , M is mass and t is time. Time

Based on what we have in equation 2.26 for 1D ow, the input mass rate for x direction shown in gure 2.7 will be:

$$\dot{m}_{ix} = \quad : u_x : dA_x \tag{2.27}$$

$dA_x$, $u_x$ and are the normal cross sectional area in x direction, velocity and density respectively.

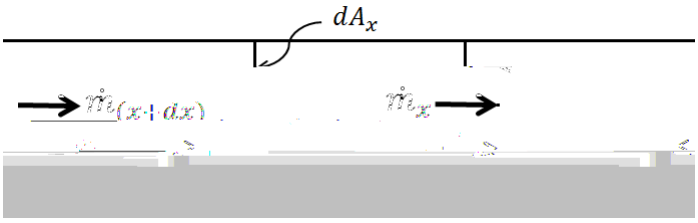



Figure 2.7: Material Balance Control Volume

Using the Euler approximation for the mass rate

$$\dot{m}_{i(x+dx)} = \dot{m}_o = \dot{m}_{i(x)} + \frac{@\dot{m}_{ix}}{@x} : dx \tag{2.28}$$

The generation and consumption terms in reservoir engineering are production and injection in wells and can be speci ed as:

$$\begin{aligned} \dot{m}_{cons} &= \quad : q_{prod} \quad ; \quad q_{prod} = \text{Production Rate} \\ \dot{m}_{gen} &= \quad : q_{inj} \quad ; \quad q_{inj} = \text{Injection Rate} \end{aligned} \tag{2.29}$$

By substituting equations 2.27 , 2.28 and 2.29 in equation 2.26:

$$\frac{@}{@x}( : u_x : dy : dz) : dx \quad ( : q_p \quad : q_i) = \frac{@}{@t}( : : : dx : dy : dz \quad ) \tag{2.30}$$

Using $V_b$ = dx : dy: dz and dividing both sides to $V_b$

Equation 2.31 is the most general type of the mass conservation law in its one dimensional form [8]. To make it more usable in reservoir engineering, Darcy law (equation 2.3) is used to substitute the velocities. Hence, the result for multi-phase ow in porous media will be

$$\frac{@}{@x} \quad \text{—} K_{abs} K_r \quad \frac{@P}{@x} \qquad \frac{}{V_b} (q_{prod} \quad q_{inj}) = \frac{@}{@t} ( \quad :S$$

$$e_{(x+dx)} = e_{ox} = \left[ k \cdot \frac{\partial T}{\partial x} + \rho u_x \cdot H \right] + \frac{\partial}{\partial x} \left[ k \cdot \frac{\partial T}{\partial x} + \rho u_x \cdot H \right] \cdot dx \; dy dz \tag{2.36}$$

$$e_{ix} \; e_{ox} = k \frac{\partial^2 T}{\partial x^2} \; \frac{\partial}{\partial x}(\rho u_x \cdot H) \; dxdydz = k \frac{\partial^2 T}{\partial x^2} \; dxdydz + \frac{\partial}{\partial x} \; -\cdot K \cdot H \cdot \frac{\partial P}{\partial x} \; dxdydz \tag{2.37}$$

For wells, the heat transfer can divided into conduction and convection based on bottom hole temperature and uid enthalpy, as below:

$$e_w = e_{cons} \; e_{gen} = (q_{inj} \; q_{prod}) H \; (2 \; kh) r_w \; \frac{\partial T}{\partial r}_{r = r_w} \tag{2.38}$$

For the accumulation term in equation 2.33, both the rock and uid must be taken into account as they both have heat capacities and are able to store energy in themselves. Therefore the accumulation term will be:

$$\frac{\partial E}{\partial t} = \frac{\partial}{\partial t}((\; _o U_o S_o + \; _w U_w S_w) \cdot ' \cdot dxdydz \; + \; _r U_r (1 \; ') \, dxdydz) \tag{2.39}$$

Now all the above equations must be com6T

supposed that two wells are located at two sides or boundaries of the reservoir. An injection well with constant rate (or total rate) and injection temperature ($T_{inj}$

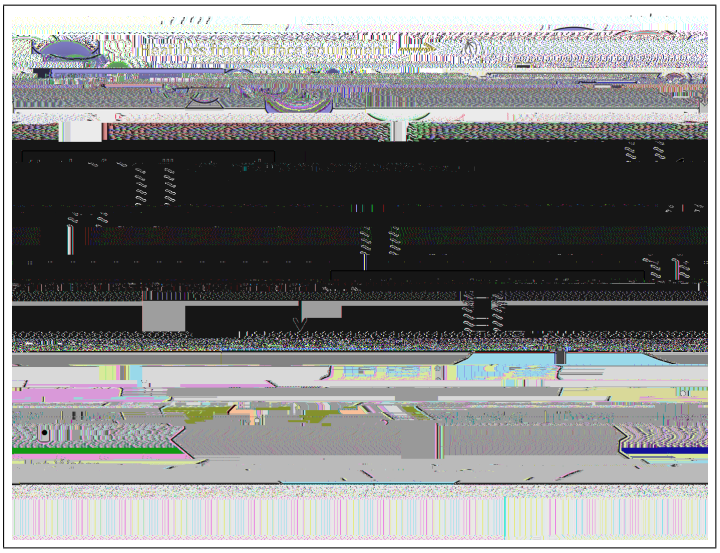


Figure 2.9: Illustration of heat losses which occur in a heat injection system

# 3 First Model

In this model, the Buckley-Leverett equation is used to find saturation profiles and then its results are used in the pressure equation which is solved by a fully implicit numerical technique. In order to find the temperature profile the saturation and pressure results are applied to a fully implicit energy equation. Finally, the mass balance equation (pressure equation) and the energy balance equation (temperature equation) are coupled to find the optimal result for pressure and temperature distributions, since these equations are highly nonlinear.

## 3.1 Buckley-Leverett Discretization

In this model, the Buckley- Leverett equation is used to find the saturation distribution. The numerical scheme used to solve this hyperbolic equation is the Lax-Wendroff scheme. The scheme is a second order finite difference method where the derivatives are approximated by differences of discrete values. An important requirement of numerical methods for such non-linear hyperbolic equations is to be in conservative form to maintain the conservation property of the equation. To derive the numerical method in conservative form we use standard finite difference discretization of the conservative form of the partial differential saturation equation, not the quasilinear form of the equation.

For a numerical scheme to be in conservation form [9] it must have the form

$$U_j^{n+1} = U_j^n - \frac{\Delta t}{\Delta X}\left[F(U_{j+\frac{1}{2}}^{n+\frac{1}{2}}) - F(U_{j-\frac{1}{2}}^{n+\frac{1}{2}})\right]; \tag{3.1}$$

where $U_j^n$ is an approximation to the cell average of the analytic function, $F(U_{j+\frac{1}{2}}^{n+\frac{1}{2}})$ is the numerical flux function, $\Delta t = t^{n+1} - t^n$, and $\Delta x = x_{j+\frac{1}{2}} - x_{j-\frac{1}{2}}$.

By using oil field units ( refer to nomenclature), the Buckley-Leverett equation will be,

$$\frac{A'}{5.615 q_t}\left[\frac{\partial S}{\partial t} + \frac{\partial f}{\partial x}\right] = 0 \tag{3.2}$$

By choosing,

$$\begin{cases} X = \frac{x}{L} \;) \quad dX = \frac{1}{L}dx \;) \quad \frac{@:::}{@x} = \frac{1}{L}\frac{@:::}{@X} \\ t_D = \frac{5:615\alpha_t}{A::L}t \;) \quad dt_D = \frac{5:615}{A::L}dt \\ \;) \quad \frac{@:::}{@t} = \frac{5:615\alpha_t}{A::L}\frac{@:::}{@t_D} \end{cases} \tag{3.3}$$

the BL equation converts into the dimensionless equation,

$$\frac{@S}{@t_D} + \frac{@f}{@X} = 0 \tag{3.4}$$

In order to drive the Lax-Wendro scheme applied to above equation to be in conservative form, for all intermediate blocks ($i = 2; : : : ; N_x \quad 1$), we start with;

$$\frac{@S}{@t_D} + \frac{@f}{@X} = 0 \;) \quad \frac{@S}{@t_D} = \quad \frac{@f}{@X} \tag{3.5}$$

and using Taylor-series expansion about $t_D$

$$S(X_i; t_D + \quad t_D) = S(X_i; t_D) + \quad t_D \frac{@S(X_i; t_D)}{@t_D} + \frac{t_D^2}{2}\frac{@^2S(X_i; t_D)}{@t_D^2} + O(\quad t_D)^3 \tag{3.6}$$

$$\begin{cases} \frac{@S}{@t_D} = \quad \frac{@f}{@X} \\ \frac{@^2S}{@t_D^2} = \quad (f_s)_{t_D} = \quad (f_{t_D})_X = \quad \frac{@f}{@S}\frac{@S}{@t_D}_X = \quad \frac{@f}{@S}(\quad f_X)_X = \quad \frac{@f}{@S}\frac{@f}{@X}_X \end{cases} \tag{3.7}$$

By substituting central di erences for space derivatives

$$\begin{cases} S_i^{n+1} = S_i^n \quad t_D \frac{f_{i+1} \quad f_{i \quad 1}}{2 \quad X} + \frac{t_D^2}{2}\frac{(\frac{@f}{@S}\cdot\frac{@f}{@X})_{i+\frac{1}{2}} \quad (\frac{@f}{@S}\cdot\frac{@S}{@X})_{i \quad \frac{1}{2}}}{X} \\ S_i^{n+1} = S_i^n \quad \frac{t_D}{x}\left[h_{i+\frac{1}{2}} \quad h_{i \quad \frac{1}{2}}\right] \end{cases} \tag{3.8}$$

: Conservative form

) By comparison:

$$h_{i+1} = \frac{1}{2}(f_{i+1} + f_i) \quad \frac{1}{2}\,_{i+\frac{1}{2}}(f_{i+1} \quad f_i) \tag{3.9}$$

where

$$_{i+\frac{1}{2}} = \begin{cases} \frac{t_D}{X} \quad \frac{@f}{@S}_i \quad \frac{t_D}{2}\frac{@f}{@S}_i \quad \text{if } S_i = S_{i+1} \\ \end{cases} \tag{3.10}$$

Similarly $h_{i-\frac{1}{2}} = \frac{1}{2}(f_i + f_{i-1}) - \frac{1}{2}(f_i - f_{i-1})$

$$\alpha_{i-\frac{1}{2}} = \begin{cases} \frac{t_D}{X}\,\frac{f_i - f_{i-1}}{S_i - S_{i-1}} & \text{if } S_i \neq S_{i-1} \\ \frac{t_D}{X}\,\frac{@f}{@S}_{i-1} & \text{if } S_i = S_{i-1} \end{cases} \tag{3.11}$$

### 3.1.1 Effect of boundary conditions

For the first cell ( $i = 1$), it is supposed that there is a known value of $S$ at the boundary ($S = 1 - S_{or}$). At this point the Lax-Wendroff scheme is derived based on the unequal spacing (see figure 3.1)

Figure 3.1: Unequal spacing for the left boundary cell after introducing an imaginary node

$$\begin{cases} f(X_1 + \Delta X) = f(X_1) + \Delta X \cdot \frac{@f}{@X}_{X_1} \\ f(X_1 - X_b) = f(X_1) - X_b \cdot \frac{@f}{@X}_{X_1} \end{cases} \Rightarrow \frac{@f}{@X}_{X_1} = \frac{f_2 - f_b}{\Delta X + X_b} \tag{3.12}$$

$$S_1^{n+1} = S_1^n - t_D \frac{f_2 - f_b}{\Delta X + X_b} + \frac{t_D^2}{2}\left[\frac{@f}{@S}\cdot\frac{@f}{@X}\right]_{1+\frac{\Delta X}{2}} S$$

where

$$1+\frac{X}{2} = \begin{cases} \frac{t_D}{X}\cdot\frac{f_2 - f_1}{S_2 - S_1} & S_2 \neq S_1 \\ \frac{t_D}{X}\cdot\frac{@f}{@S}_1 & S_2 = S_1 \end{cases}$$

$$1-\frac{X_b}{2} = \begin{cases} \frac{t_D}{X_b}\cdot\frac{f_1 - f_b}{S_1 - S_b} & S_b \neq S_1 \\ \frac{t_D}{X_b}\cdot\frac{@f}{@S}_b & S_b = S_1 \end{cases} \qquad (3.16)$$

All the fractional water values and its derivatives in above formulas are calculated from equations 2.24 and 2.25

For last cell ($i = N_x$), the equation 3.4 is discretized based on forward in time and backward in space as follows,

$$\frac{S_i^{n+1} - S_i^n}{t_D} + \frac{f(S_i^n) - f(S_{i-1}^n)}{X} = 0 \qquad (3.17)$$

$$\Rightarrow S_i^{n+1} = S_i^n - \frac{t_D}{X}$$

the stencil of the scheme, whilst the line AC violates the CFL condition, by lying outside the domain of dependence.

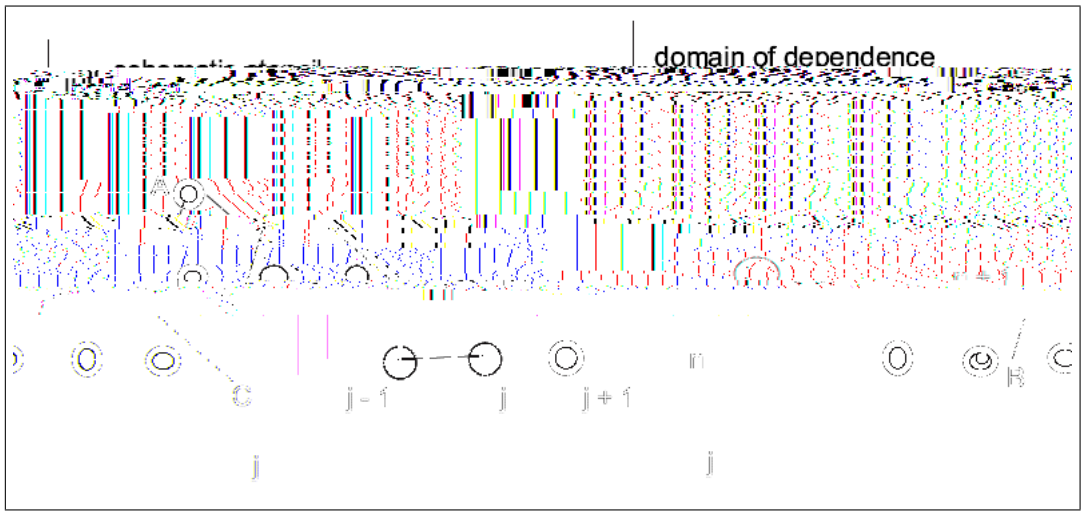


Figure 3.3 shows the grid indexing scheme used for the method. The above equation is discretized as,

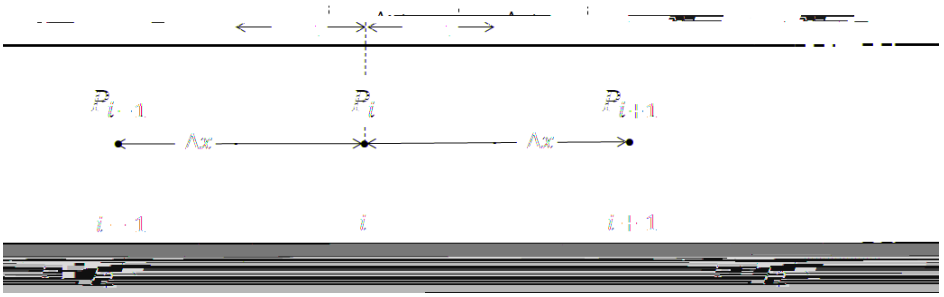



Figure 3.3: Grid indexing scheme in material balance equation

$$\frac{o\frac{K_o}{o}\frac{@p}{@x}^{n+1}_{i+\frac{1}{2}} \quad o\frac{K_o}{o}\frac{@p}{@x}^{n+1}_{i\ \frac{1}{2}}}{x} + \quad oi \quad q_o$$

$$D_{oi+1}^{n+1} p_{i+1}^{n+1} \quad p_i^{n+1} \quad D_{oi}^{n+1} p_i^{n+1} \quad p_i^{n+1}{}_1$$
$$= \frac{'}{6:328\ t} \quad 1 \quad S_i^{n+1} \quad {}^0_{opi} \; p_i^{n+1} \quad p_i^n + 1 \quad S_i^{n+1} \quad {}^0_{oTi} \; T_i^{n+1} \quad T_i^n \quad {}_{oi} \; S_i^{n+1} \quad S_i^n \tag{3.27}$$

So, general oil pressure equation for each middle cell ($i=2,\ldots,N_x \; 1$) is de ned as a function of three variable $p_{i+1}$, $p_i$ and $p_{i \; 1}$,

$$D_{oi}^{n+1} : p_i^{n+1}{}_1 \quad D_{oi}^{n+1} + D_{oi+1}^{n+1} + \frac{'}{6:328\ t} \quad 1 \quad S_i^{n+1} \quad {}^0_{opi} \; p_i^{n+1} + D_{oi+1}^{n+1} p_{i+1}^{n+1} =$$
$$\frac{'}{6:328\ t} \quad 1 \quad S_i^{n+1} \quad {}^0_{opi} : p_i^n + \frac{'}{6:328\ t} \quad 1 \quad S_i^{n+1} \quad {}^0_{oTi} \; T_i^{n+1} \tag{3.28}$$
$$T_i^n \quad \frac{'}{6:328\ t} \quad {}_{oi} \; S_i^{n+1} \quad S_i^n$$

Treating the equation for boundary cells is slightly di erent. For the rst cell ($i = 1$), the equation 3.23 is discretized as

$$\frac{o \frac{K_o}{o} \frac{@P}{@x}^{n+1}_{i+\frac{1}{2}} \quad o \frac{K_o}{o} \frac{@P}{@x}^{n+1}_{bl}}{x} + \frac{{}_{oi} \; q_o}{1:127 V_{bi}}$$
$$= \frac{'}{6:328\ t} \quad 1 \quad S_i^{n+1} \quad {}^0_{opi} \; p_i^{n+1} \quad p_i^n + 1 \quad S_i^{n+1} \quad {}^0_{oTi} \; T_i^{n+1} \quad T_i^n \quad {}_{oi} \; S_i^{n+1} \quad S_i^n \tag{3.29}$$

Using equation 2.43 and boundary conditions help to nd water properties and boundary pressure gradient in above equation,

$$S_b = 1 \quad S_{or} \; ; \; T_b = T_{inj} \; ; \; p_b = p_{bl}$$
$$K_{wbl} = K_{abs} : K_{rw} \; (1 \quad S_{or}) = K_{abs} : K_{rw}^{max}$$
$${}_{wb} = {}_w \; (P_{bl}; T_{inj}) \tag{3.30}$$
$$\frac{P_i \quad P_{bl}}{x_{/2}} = \frac{@P}{@x}_{bl} = \frac{q_t : {}_{wbl}}{1:127 K_{wbl} A_n} = GPI$$

Note that the injection well is located on the boundary, so the e ect of generation terms is considered in the boundary pressure gradient. Hence, the discretized oil pressure equation for the rst cell ($i = 1$) is

$$D_{oi+1}^{n+1} + \frac{' \quad 1 \quad S_i^{n+1} \quad {}^0_{opi}}{6:328\ t} \; p_i^{n+1} + D_{oi+1}^{n+1} : p_{i+1}^{n+1} = D_{oi}^{n+1} : \quad x : GPI$$
$$\frac{' \quad 1 \quad S_i^{n+1} \quad {}^0_{opi}}{6:328\ t} P_i^n + \frac{' \; (1 \quad S_i) \; {}^0_{oTi}}{6:328\ t} \; T_i^{n+1} \quad T_i^n \quad \frac{' \quad {}_{oi}}{6:328\ t} \; S_i^{n+1} \quad S_i^n \tag{3.31}$$

Condition at the right boundary cell is di erent. Using equation 2.44 and de ning a new parameter, $_{tb}$, the pressure gradient for this boundary is calculated as

$$_{tb} = \frac{K_{rob}}{_{ob}} + \frac{K_{rwb}}{_{wb}} \quad ) \quad \frac{@P}{@x}_{b} = \frac{q_t}{1{:}127\, K_{abs} A_n \;_{tb}} = \quad GP\,O \tag{3.32}$$

$$\text{Boundary conditions } (i = Nx) \quad \begin{cases} S_b = \frac{3}{2} S_{Nx} \quad \frac{1}{2} S_{Nx \;\; 1} \\ p_b = p_{br} \\ T_b = \frac{3}{2} T_{Nx} \quad \frac{1}{2} T_{Nx \;\; 1_x \;\; 1} \end{cases} \quad ) \quad \begin{cases} K_{rob} = K_{ro} \; (1 \quad S_b) \\ K_{rwb} = K_{rw} \; (S_b) \\ _{ob} = \;\; _{o} \; (p_b; T_b) \\ _{wb} = \;\; _{w} \; (p_b; T_b) \end{cases}$$

By substituting this term into the equation below (discretized pressure equation for the last cell),

$$\frac{o \frac{K_o}{_o} \frac{@P}{@x}^{n+1}_{br} \qquad o \frac{K_o}{_o} \frac{@P}{@x}^{n+1}_{i \;\; \frac{1}{2}}}{x} = \frac{'}{6{:}328 \quad t} \quad 1 \quad S_i^{n+1} \quad {}^{0}_{opi} \quad p_i^{n+1} \quad p_i^n + 1 \quad S_i^{n+1} \quad {}^{0}_{oTi} \quad T_i^{n+1} \quad T_i^n \qquad _{oi} \; S_i^{n+1} \quad S_i^n \tag{3.33}$$

And noting that the generation and consumption term in the main equation 3.21 is replaced by the e ect of the boundary condition, we have the equation below for $= N_x$ ,

$$D^{n+1}$$

$$F(x) = Ax \quad B = 0 \tag{3.35}$$

Jacobian. As the number of equations and unknowns, $N_x$ ; increases, so do the number of elements in the Jacobian.

The convergence of Newton's method is quadratic when the Jacobian matrix is non-singular and the initial guess is close enough.

;1614 985.se enough.

## 3.4 Jacobian Matrix De nition for Mass Balance Equation

In order to solve the nonlinear system resulted from applying fully implicit method to oil mass conservation equation (oil pressure equation), the Jacobian is de ned as follow,

For internal equations ($i = 2; : : : ; N_x$

(3.43)

$$\frac{@F_i}{@P_{i+1}} = \frac{1}{2}\ {}^{n+1}_{opi+1} D^{n+1}_{oi+1} \quad P^{n+1}_{i+1} \quad P^{n+1}_{i} + D^{n+1}_{oi+1}$$
$$\frac{1}{2} GPI: \quad x: \ {}^{n+1}_{opi} D^{n+1}_{oi} \quad \frac{1}{2} GPI: \quad x:D^{n+1}_{oi} : \frac{{}^{0}_{wpb}}{wb} \tag{3.44}$$

For the last equation ($i = N_x$)

$$\frac{@F_i}{@P_i} = \frac{1}{2} D^{n+1}_{oi} \quad {}^{n+1}_{opi} P^{n+1}_{i\ 1} \quad D^{n+1}_{oi} + \frac{' \quad 1 \quad S^{n+1}_{i} \quad {}^{0}_{opi}}{6:328 \quad t} \Bigg)$$
$$\frac{1}{2} D^{n+1}_{oi} \quad {}^{n+1}_{opi} + \frac{' \quad 1 \quad S^{n+1}_{i} \quad {}^{00}_{oppi}}{6:328 \quad t} \Bigg) P^{n+1}_{i} \quad \frac{3}{2} D^{n+1}_{oi} \quad {}^{n+1}_{opi} \quad x:GPO$$
$$+ \frac{' \quad 1 \quad S^{n+1}_{i} \quad {}^{00}_{oppi}}{6:328 \quad t} P^{n}_{i} \quad \frac{' \quad 1 \quad S^{n+1}_{i} \quad {}^{00}_{opTi}}{6:328 \quad t} \ T^{n+1}_{i} \quad T^{n}_{i}$$
$$+ \frac{' \quad {}^{0}_{opi}}{6:328 \quad t} \ S^{n+1}_{i} \quad S^{n}_{i} \tag{3.45}$$

$$\frac{@F_i}{@P_{i\ 1}} = D^{n+1}_{oi} \quad 1 + \frac{1}{2} \ {}_{opi} \ P^{n+1}_{i\ 1} \quad P^{n+1}_{i} + \frac{1}{2} D^{n+1}_{oi} \quad {}^{n+1}_{opi} \quad x:GPO \tag{3.46}$$

where

$${}^{n+1}_{opi} = \frac{{}^{0}_{op}}{o} \quad \frac{{}^{0}_{op}}{o} \Bigg)^{n+1}_{i \ \frac{1}{2}} \qquad i = 1; :::; N_x + 1 \tag{3.47}$$

Derivatives of oil and water densities and viscosities to pressure and temperature are de ned using the equations 2.9 - 2.12.

## 3.5 Well Coupling

The right boundary well pressure equation is

$$q_t = \frac{1:127K_{abs}A_n}{x=2}\quad \frac{K_{rw}}{_w(P_{wr})} + \frac{K_{ro}}{_o(P_{wr})}\quad (P_{wr}\quad P_B) \tag{3.49}$$

## 3.6 Energy Balance Equation Discretization

The energy equation 2.40 in oil eld units ( refer to nomenclature) is rewritten as

$$\begin{aligned}&\frac{@}{@x}:(24K_H\frac{@T}{@x}) + 6:328\frac{@}{@x}:f(\ _o\frac{K_o}{_o}H_o + \ _w\frac{K_w}{_w}H_w)\frac{@P}{@x}g + \frac{e}{Vb}\\ &\quad = \frac{@}{@t}f'(\ _oS_oU_o + \ _wS_wU_w) + (1\quad ')\ _rU_rg\end{aligned} \tag{3.50}$$

$$\begin{aligned}H\quad &= H^{ref} + C_P\ (T\quad T_{ref}) &&= o;w\\ U\quad &= U^{ref} + C_V\ (T\quad T_{ref}) &&= o;w;r\end{aligned} \tag{3.51}$$

A fully implicit central nite di erence scheme is used to discretize the equation. First, the equation is expanded by substituting equation 3.51 and then new equation is shortened by de ning some parameters,

$$\begin{aligned}&\frac{@}{@x}:(24K_H\frac{@T}{@x}) + 6:328\frac{@}{@x}:f(\ _o\frac{K_o}{_o}(H_o^{ref} + C_{Po}(T\quad T_{ref}))\frac{@P}{@x}g\\ &+ 6:328\frac{@}{@x}f\ _w\frac{K_w}{_w}(H_w^{ref} + C_{Pw}(T\quad T_{ref}))\frac{@P}{@x}g + \frac{e}{Vb} =\\ &\quad \frac{@}{@t}f'(\ _oS_o(U_o^{ref} + C_{Vo}(T\quad T_{ref})) + \ _wS_w(U_w^{ref} + C_{Vw}(T\quad T_{ref})))\\ &\quad + (1\quad ')\ _r(U_r^{ref} + C_{Vr}(T\quad T_{ref}))g\end{aligned} \tag{3.52}$$

By de ning,

$$HR = \ _o\frac{K_o}{}$$

$$\frac{@}{@x}:(24K_H\frac{@T}{@x}) + 6:328\frac{@}{@x}((HR + HB:T)\frac{@P}{@x}) + \frac{P}{V_b}e = \frac{@}{@t}(UR + UB \quad T) \tag{3.57}$$

### 3.6.1 Discretization of Right Hand Side of the Energy Equation 3.57

Replace the derivatives as follow

$$\begin{aligned}\frac{@}{@t}(UR + UB:T) &= \frac{1}{t}f(UR + UB:T)^{n+1} \quad (UR + UB:T)^n g \\ &= (\frac{UR^{n+1} \quad UR^n}{t}) + (\frac{UB^{n+1}}{t}):T^{n+1} \quad (\frac{UB^n}{t})T^n\end{aligned} \tag{3.58}$$

### 3.6.2 Discretization of Left Hand Side of Energy Equation 3.57 for Middle Cells ( $i = 2;::::;N_x$ )

This side of the equation is divided into two terms; conductive and convective heat transfer. Conduction is the transfer of heat energy by di usion due to the temperature gradient. In this project, conduction takes place in both rock and uids. While convective heat transfer takes place through advection mostly, in which heat is transferred by the motion of currents in the uid.

#### 3.6.2.1 Conduction Term

In this dissertation, conductive heat is considered to transfer in two dimensions in order to model heat loss to adjacent strata [11]. Figure 3.4 shows the schematic diagram of the model.

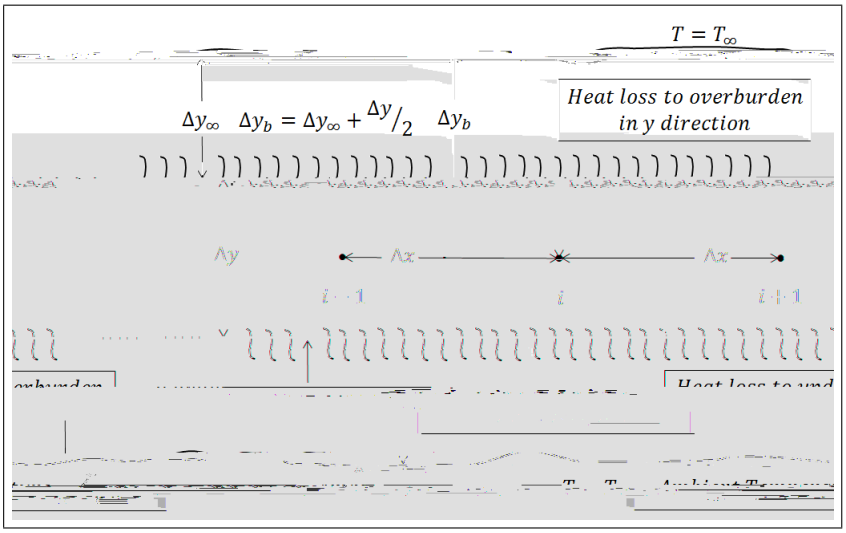



Figure 3.4: Heat transfer in the x and y direction by conduction

Using a central di erence scheme with equal spacing in the x direction and unequal spacing in the y direction results in

$$\frac{@}{@x}(24K_H\frac{@T}{@x}) + \frac{@}{@y}(24K_r\frac{@T}{@Y}) =$$

$$\frac{(24K_H\frac{@T}{@x})^{n+1}_{i+\frac{1}{2}} \quad (24K_H\frac{@T}{@x})^{n+1}_{i\ \frac{1}{2}}}{x} + \frac{(24k_r\frac{@T}{@y})^{n+1}_{UP} \quad (24K_r\frac{@T}{@y})^{n+1}_{Down}}{y_1 + \frac{y}{2}} = \tag{3.59}$$

$$(\frac{24K_H}{x^2})T_i^{n+1}{}_1 \quad (\frac{48K_H}{x^2} + \frac{48k_r}{y_b^2})T_i^{n+1} + (\frac{24K_H}{x^2})T_{i+1}^{n+1} + (\frac{48K_r}{y_b^2})T_1$$

where $y_b = y_1 + \frac{y}{2}$.

Convection term is sorted out as

$$6{:}328\frac{@}{@x}((HR + HB{:}T)\frac{@P}{@X}) =$$
$$\frac{6{:}328}{x}(HR_{i+1}^{n+1} : GP_{i+1}^{n+1} \quad HR_i^{n+1} : GP_i^{n+1}) \quad \frac{6{:}328}{2\ x}(HB_i^{n+1} : GP_i^{n+1})\,T_i^{n+1} + \qquad (3.61)$$
$$\frac{6{:}328}{2\ x}(HB_{i+1}^{n+1} : GP_{i+1}^{n+1} \quad HB_i^{n+1} : GP_i^{n+1})\,T_i^{n+1} + \frac{6{:}328}{2\ x}(HB_{i+1}^{n+1} : GP_{i+1}^{n+1})\,T_i^{n+1}$$

By combining equations 3.57, 3.58, 3.59 and 3.61 together, the general energy equation for middle cells is obtained as( $i = 2; : : : ; N_x$);

$$f\frac{24K_H}{x^2} \quad \frac{6{:}328}{2\ x}(HB_i^{n+1} : GP_i^{n+1})gT_i^{n+1} f\frac{24K_H}{x^2} \quad \frac{6{:}328}{2\ x}(HB_i^{n+1} : GP_i^{n+1})gT_i^{n+1}$$
$$+ f \quad \frac{48K_H}{x^2} \quad \frac{48k_r}{y_b^2} + \frac{6{:}328}{2\ x}(HB_{i+1}^{n+1} : GP_{i+1}^{n+1} \quad HB_i^{n+1} : GP_i^{n+1}) \quad \frac{UB_i^{n+1}}{t}gT_i^{n+1}$$
$$+ f\frac{24K_H}{x^2} + \frac{6{:}328}{2\ x}(HB_{i+1}^{n+1} : GP_{i+1}^{n+1}gT_{i+1}^{n+1} = (\frac{UR_i^{n+1} \quad UR_i^n}{t})$$
$$(\frac{UB_i^n}{t})T_i^n \quad (\frac{48K_r}{y_b^2})T_1 \quad \frac{6{:}328}{x}(HR_{i+1}^{n+1} : GP_{i+1}^{n+1} \quad HR_i^{n+1} : GP_i^{n+1}) \qquad (3.62)$$

### 3.6.3 Calculations for the Left Boundary Cell (i =1)

For this cell, the conduction and convection terms are treated di erently because of the e ect of boundary conditions.

#### 3.6.3.1 Conduction Term

Applying a central di erence scheme to the conduction term in the x direction (with unequal spacing) gives;

$$\frac{@}{@x}(24K_H\frac{@T}{@x}) = \frac{24K_H(\frac{T_{i+1}^{n+1} \quad T_i^n}{x}) \quad 24K_H(\frac{T_i^{n+1} \quad T_{inj}}{x=2})}{(\frac{3\ x}{2})=2} \qquad (3.63)$$

and in the y direction

$$\frac{@}{@y}(24K_r\frac{@T}{@y}) = \frac{24k_r(\frac{T_1 \quad T_i^{n+1}}{y_1 + \ y=2}) \quad 24K_r(\frac{T_i^{n+1} \quad T_1}{y_1 + \ y=2})}{(\ y_1 + \ y=2)} \qquad (3.64)$$

By adding these two together and rearranging the terms, the conduction term is written as

$$\frac{@}{@x}(24K_H\frac{@T}{@x}) + \frac{@}{@y}(24K_r\frac{@T}{@y}) = \frac{@}{@x}(24K_H\frac{@T_i}{@x}) + \frac{@}{@y}(24K_r\frac{@T_i}{@y}) =$$

$$(\frac{96K_H}{x^2} \quad \frac{48K_r}{y_b^2})T_i^{n+1} + (\frac{32K_H}{x^2})T_{i+1}^{n+1} + (\frac{64K_H}{x^2})T_{inj} + (\frac{48K_r}{y_b^2})T_1 \tag{3.65}$$

3.6.3.2 Convection Term

By expanding the convection term on the first cell and substituting the injection temperature,

$$6:328\frac{@}{@x}((HR + HB:T)\frac{@P}{@x}) =$$

$$\frac{6:328}{x}\{HR:GP + HB:GP:T\}_{i+\frac{1}{2}}^{n+1} \quad \frac{6:328}{x}\{HR:GP + HB:GP:T\}_b^{n+1} =$$

$$(\frac{3:164}{x})(HB_{i+1}^{n+1}:GP_{i+1}^{n+1})(T_i^{n+1} + T_{i+1}^{n+1}) + (\frac{6:328}{x})\{HR_{i+1}^{n+1}:GP_{i+1}^{n+1} \quad HR_i^{n+1}:GP_i^{n+1}\}$$

$$(\frac{6:328}{x})(HB_i^{n+1}:GP_i^n)T_{inj} \tag{3.66}$$

$$6{:}328\frac{@}{@x}((HR + HB{:}T)\frac{@P}{@x}) = \frac{6{:}328}{x}(HR_{i+1}^{n+1} : GP_{i+1}^{n+1} \quad HR_{i}^{n+1} : GP_{i}^{n+1})$$
$$+ (\frac{3{:}164}{x})(3{:}HB_{i+1}^{n+1} : GP_{i+1}^{n+1} \quad HB_{i}^{n+1} : GP_{i}^{n+1}){:}T_{i}^{n+1}$$
$$(\frac{3{:}164}{x})(HB_{i+1}^{n+1} : GP_{i+1}^{n+1} + HB_{i}^{n+1} : GP_{i}^{n+1}){:}T_{i}^{n+1}$$

Based on above de nitions and equation 3.62, the Jacobian calculations for $i = 2;::::;N_x \quad 1$ give

$$\frac{@F_i}{@T_{i\ 1}} = (\frac{24K_H}{x^2} \quad \frac{3:164}{x}(HB_i^{n+1} : GP_i^{n+1})) \quad 3:164 \quad x: DHR_i^{n+1} : GP_i^{n+1}$$

$$3:164(\frac{x}{2}): DHB_i^{n+1} : GP_i^{n+1} :(T_i^{n+1} + T_{i\ 1}^{n+1}$$

$$\frac{@F_i}{@T_i} = f\left(\frac{3:164}{x}\right)(3HB_{i+1}^{n+1} : GP_{i+1}^{n+1} \quad HB_i^{n+1} : GP_i^{n+1}) \quad \left(\frac{36K_r}{x: \ x_1}\right) \quad \left(\frac{48K_r}{y_b^2}\right) \quad \frac{UB_i^{n+1}}{t} g$$

$$3:164\left(\frac{x}{2}\right)(3\,DHB_{i+1}^{n+1} : GP_{i+1}^{n+1} + DHB_i^{n+1} : GP_i^{n+1}) : T_i^{n+1}_1$$

$$+ f\,3:164\left(\frac{x}{2}\right)(9\,DHB_{i+1}^{n+1} : GP_{i+1}^{n+1} \quad DHB_i^{n+1} : GP_i^{n+1}) \quad \frac{DUB_i^{n+1}}{t} g : T_i^{n+1}$$

$$+ 3:164: \quad x:(3\,DHR_{i+1}^{n+1} : GP_{i+1}^{n+1} \quad DHR_i^{n+1} : GP_i^{n+1}) \quad \frac{DUR_i^{n+1}}{t}$$

(3.82)

## 3.7 Summary of The First Model

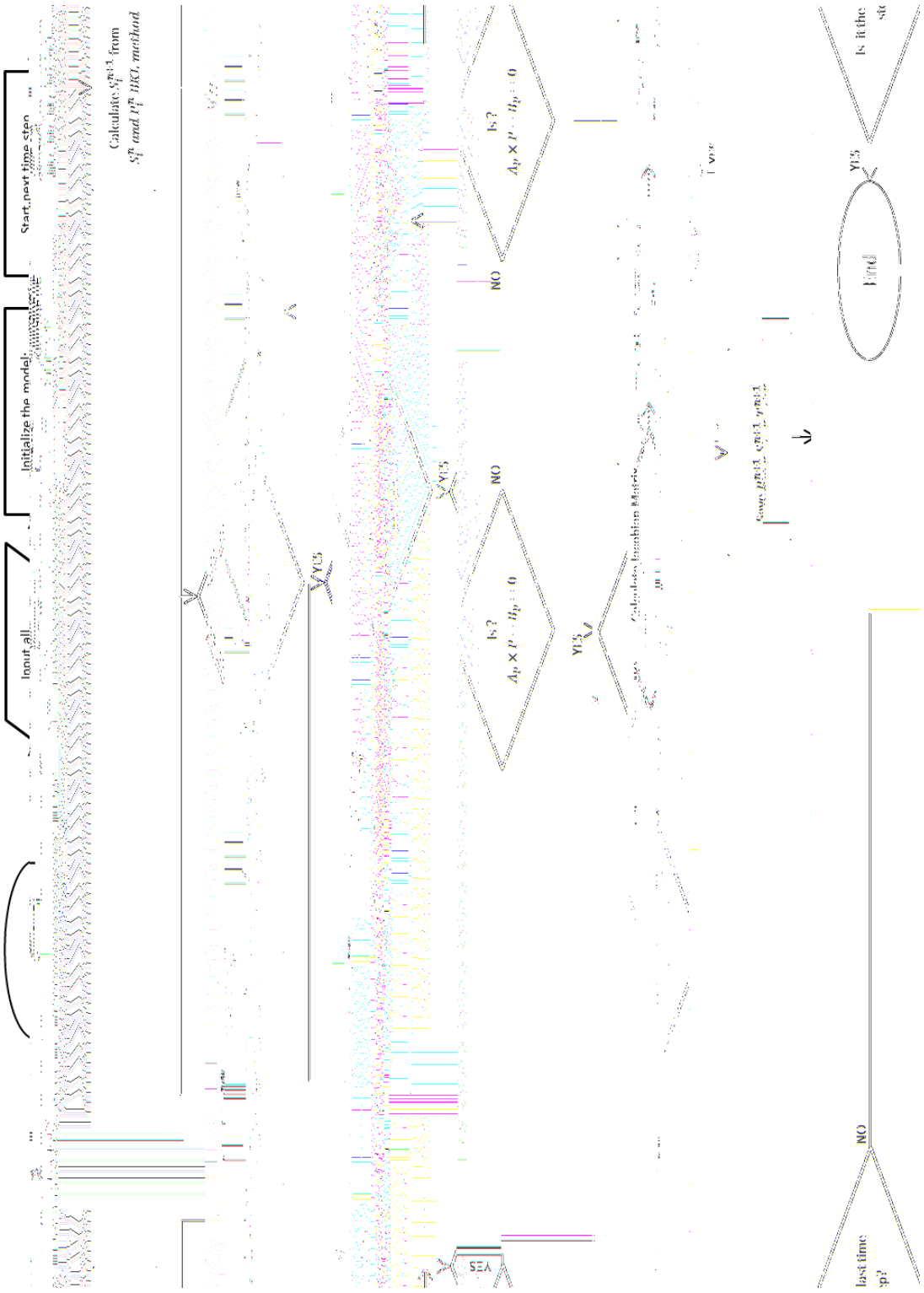



Figure 3.5: The First Model Calculations Flow Chart

# 4 Second Model

In this model, the pressure will be solved implicitly and after finding the pressure solution, saturation values can be determined explicitly. This technique is called IMPES and is much used in the oil industry [8]. During one time step the results of IMPES are used in the temperature equation which is solved fully implicitly, and finally there will a coupling between the IMPES technique and the fully implicit temperature equation in order to find the final pressure, saturation and temperature distribution results.

## 4.1 IMPES Technique

In the oil and water system, the general 3D equations for oil and water are

$$\begin{cases} r : \; {}_o\frac{K_o}{{}_o}\tilde{r}P_o + \frac{{}_o q_o}{1{:}127V_b} = \frac{1}{6{:}328}\frac{@}{@t}({}_o{}'S_o) \\ r : \; {}_w\frac{K_w}{{}_w}\tilde{r}P_w + \frac{{}_w q_w}{1{:}127V_b} = \frac{1}{6{:}328}\frac{@}{@t}({}_w{}'S_w) \end{cases} \tag{4.1}$$

By considering the assumptions made in section 2.1 , and considering that generation and consumption terms are replaced by boundary conditions and expansion of right hand side of the equations, we have

$$\begin{cases} \frac{@}{@x}\left[{}_o\frac{K_o}{{}_o}\frac{@P}{@x}\right] = \frac{'}{6{:}328}\left[(1-S)\,{}^0_{op}\frac{@P}{@t} + (1-S)\,{}^0_{oT}\frac{@T}{@t} - {}_o\frac{@S}{@t}\right] \\ \frac{@}{@x}\left[{}_w\frac{K_w}{{}_w}\frac{@P}{@x}\right] = \frac{'}{6{:}328}\left[S\,{}^0_{wp}\frac{@P}{@t} + S\,{}^0_{wT}\frac{@T}{@t} - {}_w\frac{@S}{@t}\right] \end{cases} \tag{4.2}$$

Divide oil pressure equation by ${}_o$ and water pressure equation by ${}_w$ , then add them together, and rearrange the final equation to find the discretized pressure equation for internal cells ($i = 2;\ldots;N_x - 1$), giving

$$\begin{aligned} &\frac{1}{{}_o}\frac{@}{@x}\left[{}_o\frac{K_o}{{}_o}\frac{@P}{@x}\right] + \frac{1}{{}_w}\frac{@}{@x}\left[{}_w\frac{K_w}{{}_w}\frac{@P}{@x}\right] = \\ &\qquad \frac{'}{6{:}328}\left[(1-S)\frac{{}^0_{op}}{{}_o} + S\frac{{}^0_{wp}}{{}_w}\right]\frac{@P}{@t} + \frac{'}{6{:}328}\left[(1-S)\frac{{}^0_{oT}}{{}_o} + S\frac{{}^0_{wT}}{{}_w}\right]\frac{@T}{@t} \end{aligned} \tag{4.3}$$

$$
\begin{aligned}
&\left(\frac{D_{oi}^{n+1}}{oi}+\frac{D_{wi}^{n+1}}{wi}\right)P_i^{n+1} - \left\{\frac{D_{oi}^{n+1}}{oi}+\frac{D_{wi}^{n+1}}{wi}+\frac{\phi(1-S_i^n)}{6.328\,t}\left(\frac{\partial^0 o_{pi}}{oi}\right)+\frac{\phi S_i^n}{6.328\,t}\left(\frac{\partial^0 w_{pi}}{wi}\right)\right\}P_i^{n+1} = \\
&\quad GPO:\ x:\left(\frac{D_{oi+1}^{n+1}}{oi}+\frac{D_{wi+1}^{n+1}}{wi}\right) - \left\{\frac{\phi(1-S_i^n)}{6.328\,t}\left(\frac{\partial^0 o_{pi}}{oi}\right)+\frac{\phi S_i^n}{6.328\,t}\left(\frac{\partial^0 w_{pi}}{wi}\right)\right\}P_i^n \\
&\quad - \left\{\frac{\phi(1-S_i^n)}{6.328\,t}\left(\frac{\partial^0 o_{Ti}}{oi}\right)+\frac{\phi S_i^n}{6.328\,t}\left(\frac{\partial^0 w_{Ti}}{wi}\right)\right\}(T_i^{n+1}-T_i^n) \\
&\quad - \left\{\frac{\phi(1-S_i^n)}{6.328\,t}\left(\frac{\partial^0 o_{Ti}}{oi}\right)+\frac{\phi S_i^n}{6.328\,t}\left(\frac{\partial^0 w_{Ti}}{wi}\right)\right\}(T_i^{n+1}-T_i^n)
\end{aligned}
\tag{4.8}
$$

The result of writing the pressure equation for all blocks is also a nonlinear system to solve, so the de nition of the Jacobian for this system is required.

## 4.2 Jacobian Calculations for the Pressure Equation

The general equation 4.4 is a function of three variables; $P_{i-1}$; $P_i$ and $P_{i+1}$, therefore the Jacobian matrix is de ned as follow

$$
\frac{\partial F_i}{\partial P_{i-1}} = \frac{1}{2}\left(\frac{D_{wi}^{n+1}}{wi}\,\frac{\partial^{n+1} w_{pi}}{wi}+\frac{D_{oi}^{n+1}}{oi}\,\frac{\partial^{n+1} o_{pi}}{oi}\right)(P_{i-1}^{n+1}-P_i^{n+1})+\left(\frac{D_{oi}^{n+1}}{oi}+\frac{D_{wi}^{n+1}}{wi}\right) \tag{4.9}
$$

$$
\begin{aligned}
\frac{\partial F_i}{\partial P_i} &= \left\{D_{oi+1}^{n+1}\left(\frac{1}{2}\frac{\partial^{n+1} o_{pi+1}}{oi}-\frac{\partial^0 o_{pi}}{2\,oi}\right)+D_{wi+1}^{n+1}\left(\frac{1}{2}\frac{\partial^{n+1} w_{pi+1}}{wi}-\frac{\partial^0 w_{pi}}{2\,wi}\right)\right\}P_{i+1}^{n+1} \\
&\quad - \left\{\frac{D_{oi}^{n+1}+D_{oi+1}^{n+1}}{oi}+\frac{D_{wi}^{n+1}+D_{wi+1}^{n+1}}{wi}+\frac{\phi(1-S_i^n)}{6.328\,t}:\frac{\partial^0 o_{pi}}{oi}+\frac{\phi S_i^n}{6.328\,t}:\frac{\partial^0 w_{pi}}{wi}\right\} \\
&\quad - \left\{\frac{D_{oi}^{n+1}\,\partial^{n+1} o_{pi}+D_{oi+1}^{n+1}\,\partial^{n+1} o_{pi+1}}{2\,oi}+\frac{D_{wi}^{n+1}\,\partial^{n+1} w_{pi}+D_{wi+1}^{n+1}\,\partial^{n+1} w_{pi+1}}{2\,wi}\right. \\
&\quad \left.(D_{oi}^{n+1}+D_{oi+1}^{n+1})\left(\frac{\partial^0 o_{pi}}{oi}\right)-(D_{wi}^{n+1}+D_{wi+1}^{n+1})\left(\frac{\partial^0 w_{pi}}{wi}\right)+\frac{\phi(1-S_i^n)}{6.328\,t}:\frac{\partial^{00} o_{ppi}\,oi-(\partial^0 o_{pi})^2}{2\,oi}\right. \\
&\quad \left.+\frac{\phi S_i^n}{6.328\,t}:\frac{\partial^{00} w_{ppi}\,wi-(\partial^0 w_{pi})^2}{2\,wi}\right\}P_i+\left\{D_{oi}^{n+1}\left(\frac{1}{2}\frac{\partial^{n+1} o_{pi}}{oi}-\frac{\partial^0 o_{pi}}{2\,oi}\right)+D_{wi}^{n+1}\left(\frac{1}{2}\frac{\partial^{n+1} w_{pi}}{wi}-\frac{\partial^0 w_{pi}}{2\,wi}\right)\right\}P_{i-1}^{n+1} \\
&\quad + \left\{\frac{\phi(1-S_i^n)}{6.328\,t}:\frac{\partial^{00} o_{ppi}\,oi-(\partial^0 o_{pi})^2}{2\,oi}+\frac{\phi S_i^n}{6.328\,t}:\frac{\partial^{00} w_{ppi}\,wi-(\partial^0 w_{pi})^2}{2\,wi}\right\}P_i^n \\
&\quad - \left\{\frac{\phi(1-S_i^n)}{6.328\,t}:\frac{\partial^{00} o_{pTi}\,oi-\partial^0 o_{pi}\,\partial^0 o_{Ti}}{2\,oi}+\frac{\phi S_i^n}{6.328\,t}:\frac{\partial^{00} w_{pTi}\,wi-\partial^0 w_{pi}\,\partial^0 w_{Ti}}{2\,wi}\right\}(T_i^{n+1}-T_i^n)
\end{aligned}
\tag{4.10}
$$

$$
\frac{\partial F_i}{\partial P_{i+1}} = \frac{1}{2}\left(\frac{D_{wi+1}^{n+1}}{wi}\,\frac{\partial^{n+1} w_{pi+1}}{wi}+\frac{D_{oi+1}^{n+1}}{oi}\,\frac{\partial^{n+1} o_{pi+1}}{oi}\right)(P_{i+1}^{n+1}-P_i^{n+1})+\left(\frac{D_{oi+1}^{n+1}}{oi}+\frac{D_{wi+1}^{n+1}}{wi}\right) \tag{4.11}
$$

The  rst cell (i = 1) pressure equation depends on two variables; $p_i$ and $p_{i+1}$ , so

$$\frac{@F_i}{@P_i} = f D_{oi+1}^{n+1}\left(\frac{1}{2}\frac{{}^{n+1}_{opi+1}}{oi} \quad \frac{{}^{0}_{opi}}{{}^{2}_{oi}}\right) + D_{wi+1}^{n+1}\left(\frac{1}{2}\frac{{}^{n+1}_{wpi+1}}{wi} \quad \frac{{}^{0}_{wpi}}{{}^{2}_{wi}}\right)g(P_{i+1}^{n+1} \quad P_i^{n+1})$$

$$f \quad \frac{' (1 \quad S_i^n)}{6{:}328 \quad t} : \frac{{}^{00}_{oppi} \quad oi \quad ({}^{0}_{opi})^2}{{}^{2}_{oi}} + \frac{' \; S_i^n}{6{:}328 \quad t} : \frac{{}^{00}_{wppi} \quad wi \quad ({}^{0}_{wpi})^2}{{}^{2}_{wi}} g(P_i^{n+1} \quad P_i^n)$$

$$f \quad \frac{D_{oi+1}^{n+1}}{oi} + \frac{D_{wi+1}^{n+1}}{wi} + \frac{' (1 \quad S_i^n)}{6{:}328 \quad t} : \frac{{}^{0}_{opi}}{oi} + \frac{' \; S_i^n}{6{:}328 \quad t} : \frac{{}^{0}_{wpi}}{wi} g$$

$$+ GPI: \quad x{:}f D_{oi}^{n+1} : \left(\frac{3}{2}\frac{{}^{n+1}_{opi}}{oi}\right.$$

## 4.3 Saturation Calculations

Oil and water saturations are evaluated explicitly by using the results of the fully implicit pressure equation. The following equations are applied to find the saturations

$$\begin{aligned}\frac{\partial}{\partial x}\left(\lambda_o \frac{K_o}{\mu_o}\frac{\partial P}{\partial x}\right) &= \frac{1}{6.328}\frac{\partial}{\partial t}\left(\phi S_o\right)\\ \frac{\partial}{\partial x}\left(\lambda_w \frac{K_w}{\mu_w}\frac{\partial P}{\partial x}\right) &= \frac{1}{6.328}\frac{\partial}{\partial t}\left(\phi S_w\right)\end{aligned} \tag{4.16}$$

These equations are discretized as

$$\begin{aligned}D^{n+1}_{oi+1}:(P^{n+1}_{i+1} - P^{n+1}_i) - D^{n+1}_{oi}(P^{n+1}_i - P^{n+1}_i) &= \frac{\Delta}{6.328}\left\{\lambda^{n+1}_{oi}:S^{n+1}_{oi} - \lambda^{n}_{oi}S^{n}_{oi}\right\}\\ D^{n+1}_{wi+1}:(P^{n+1}_{i+1} - P^{n+1}_i) - D^{n+1}_{wi}(P^{n+1}_i - P^{n+1}_i) &= \frac{\Delta}{6.328}\left\{\lambda^{n+1}_{wi}:S^{n+1}_{wi} - \lambda^{n}_{wi}S^{n}_{wi}\right\}\end{aligned} \tag{4.17}$$

Hence, oil and water saturations are calculated for internal cells ($i = 2;::::;N_x$) from

$$\begin{aligned}S^{n+1}_{oi} &= \left(\frac{6.328\ \Delta t}{\phi^{n+1}_{oi}}\right)\left\{D^{n+1}_{oi+1}:P^{n+1}_{i+1} - (D^{n+1}_{oi} + D^{n+1}_{oi+1})P^{n+1}_i + D^{n+1}_{oi}:P^{n+1}_i\right\} + \left(\frac{\lambda^{n}_{oi}}{\lambda^{n+1}_{oi}}\right)S^{n}_{oi}\\ S^{n+1}_{wi} &= \left(\frac{6.328\ \Delta t}{\phi^{n+1}_{wi}}\right)\left\{D^{n+1}_{wi+1}:P^{n+1}_{i+1} - (D^{n+1}_{wi} + D^{n+1}_{wi+1})P^{n+1}_i + D^{n+1}_{wi}:P^{n+1}_i\right\} + \left(\frac{\lambda^{n}_{wi}}{\lambda^{n+1}_{wi}}\right)S^{n}_{wi}\end{aligned} \tag{4.18}$$

For the first cell ($i = 1$),

$$\begin{aligned}S^{n+1}_{oi} &= \left(\frac{6.328\ \Delta t}{\phi^{n+1}_{oi}}\right)\left\{D^{n+1}_{oi+1}(P^{n+1}_{i+1} + P^{n+1}_i) + GPI:\ x:D^{n+1}_{oi}\right\} + \left(\frac{\lambda^{n}_{oi}}{\lambda^{n+1}_{oi}}\right)S^{n}_{oi}\\ S^{n+1}_{wi} &= \left(\frac{6.328\ \Delta t}{\phi^{n+1}_{wi}}\right)\left\{D^{n+1}_{wi+1}(P^{n+1}_{i+1} + P^{n+1}_i) + GPI:\ x:D^{n+1}_{wi}\right\} + \left(\frac{\lambda^{n}_{wi}}{\lambda^{n+1}_{wi}}\right)S^{n}_{wi}\end{aligned} \tag{4.19}$$

and finally for the right boundary cell ($i = N_x$)

$$\begin{aligned}S^{n+1}_{oi} &= \left(\frac{6.328\ \Delta t}{\phi^{n+1}_{oi}}\right)\left\{D^{n+1}_{oi}(P^{n+1}_i - P^{n+1}_{i-1}) + GPO:\ x:D^{n+1}_{oi+1}\right\} + \left(\frac{\lambda^{n}_{oi}}{\lambda^{n+1}_{oi}}\right)S^{n}_{oi}\\ S^{n+1}_{wi} &= \left(\frac{6.328\ \Delta t}{\phi^{n+1}_{wi}}\right)\left\{D^{n+1}_{wi}(P^{n+1}_i - P^{n+1}_{i-1}) + GPO:\ x:D^{n+1}_{wi+1}\right\} + \left(\frac{\lambda^{n}_{wi}}{\lambda^{n+1}_{wi}}\right)S^{n}_{wi}\end{aligned} \tag{4.20}$$

This is the whole procedure of the IMPES method for this model.

In summary, in this model pressure and saturations are calculated from the IMPES technique, but a similar method (fully implicit method) with the first model is used to find the temperature distribution.

## 4.4 Summary of the Second Model

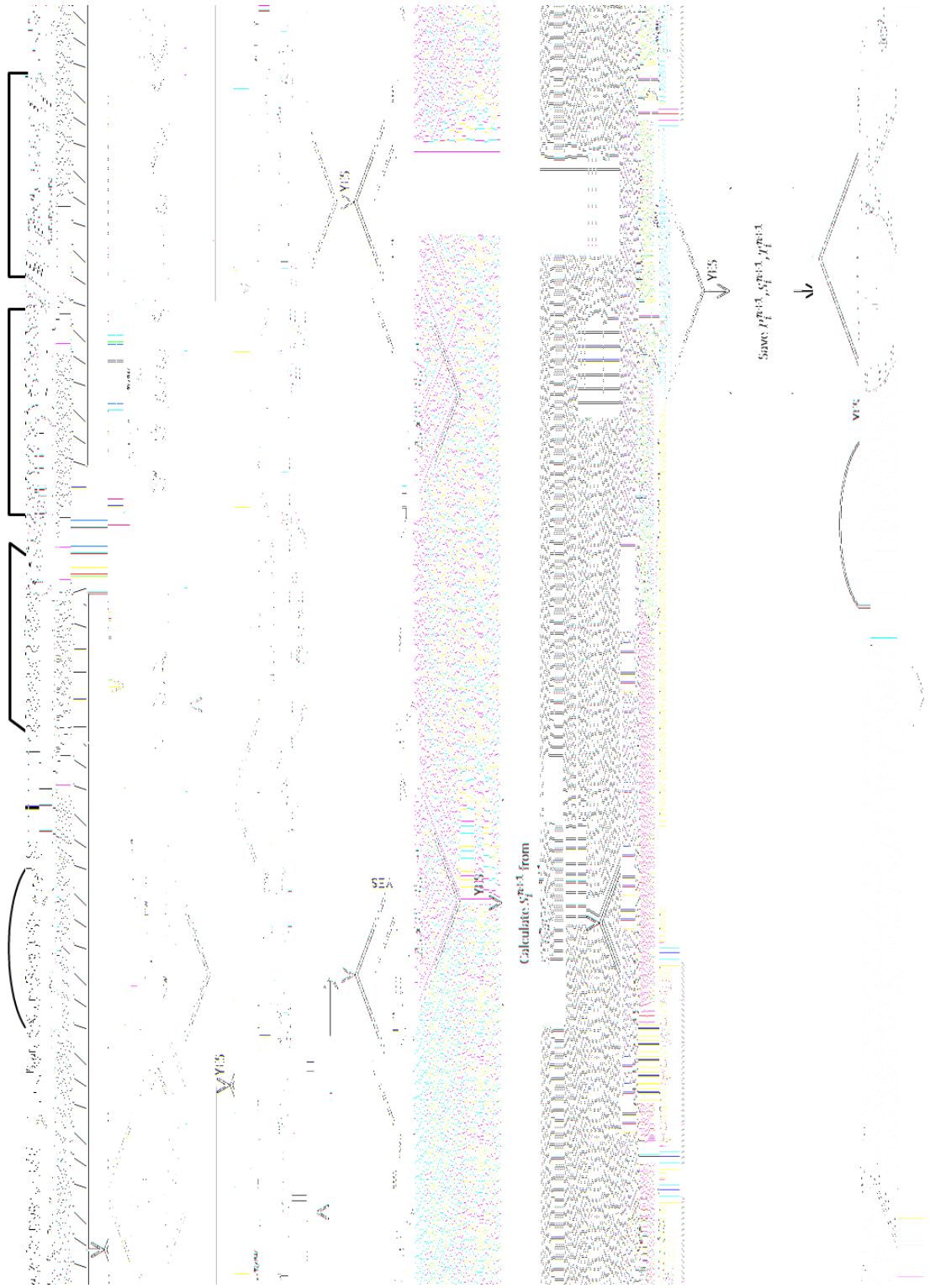


Figure 4.1: The Second Model Calculations Flow Chart

# 5 Results

It is interesting to see the results of the two di erent numerical techniques applied to a physical process and to see how choosing between these di erent techniques can change the results using the same inputs. Table 1 shows the values of the model parameters used in the two models.

## 5.1 First Model Results

In this model, the Buckley-Leverett equation and mass and energy balance equations are solved using the Lax-Wendro scheme and a fully implicit central schemes, respectively, in order to nd saturation, pressure and temperature distribution in the one dimensional hot water model. The results for pressure, saturation and temperature pro les are shown in gure 5.1 using a step size of $4x = 5(ft)$ and a time step of $4t = 0:025(day)$. Distributions are plotted after 100, 250 and 400 days. It can be seen from gure 5.1 (the water saturation pro le) that initially there is no steep front in the system but later, due to injection a shock (water front) is created which moves to the right in time. The pressure pro le changes based on the water and oil properties and shock position. Surprisingly, there are no oscillations around the discontinuity (water saturation front) although the second order accurate Lax-Wendero scheme is used to solve Buckley-Leverett equation. Figure 5.2 shows that with higher number of divisions the front is steeper, as expected, but still no oscillation is observed around this steep front. This behavior might be related to using Lax-Wendro for unequal spacing and backward in space and forward in time schemes at the boundaries or front is not steep enough.

## 5.2 Second Model Results

The fully implicit pressure explicit saturation (IMPES) and fully implicit temperature tech-

## 5.3 Comparing Two Models

By looking at the results of the two models it is clear that both have similar trends for P, S and T. Whereas, by looking at the results more closely, it can be seen that the position of steep front in the first model lag behind its corresponding front position in second model, which could be a result of using the incompressible fluid assumption in the Buckley-leverett equation for the saturation profile. Consequently lower pressure values result in the first model (lower injected fluid lower pressure increase). Therefore, the second model using IMPES technique for solving pressure and saturation and fully implicit method for temperature is a better numerical model. The only problem is that it has the numerical instability in the IMPES technique which comes from using an explicit saturation calculation. So, to make the model more applicable, the stability limits must be considered. In the next section some sensitivity analysis is performed to find these limits.

## 5.4 Sensitivity Analysis

Sensitivity analysis is performed on the second model to investigate the influence of grid and

Table 1: Model Properties

| Model properties | Value (Units) |
|---|---|
| Length (L) | 150 (ft) |
| Width (dy) | 20 (ft) |
| Height (dz) | 30 (ft) |
| Distance to ambient temperature in x direction(4 $x_1$ ) | 100 (ft) |
| Distance to ambient temperature in x direction (4 $y_1$ ) | 100 (ft) |
| Porosity (' ) | 0.2 |
| Absolute permeability ($K_{abs}$) | 3 (darcy) |
| Initial temperature (Tinit) | 559.67 ($^oR$) |
| Initial pressure (Pinit) | 4000 (psia) |
| Initial oil saturation ($S_o$init) | 0.84 |
| Total rate ($q_t$) | 1 (bbl/day) |
| Reference temperature (Tref) | 536.4 ($^oR$) |
| Ambient temperature ($T_1$ ) | 559.67 ($^oR$) |
| Injection temperature ($T_{inj}$ ) | 800 ($^oR$) |
| Rock density ( $_r$) | 145 ($lb/ft^3$) |
| Rock thermal conductivity ($k_r$) | 0.9824 (Btu/ft.hr.$^oF$) |
| Average thermal conductivity, rock, oil and water ($k_H$) | 0.4623 (Btu/ft.hr.$^oF$) |
| Oil speci c gravity (Sgo) | 0.9 |
| Water speci c heat capacity, constant pressure ($C_{pw}$) | 0.986 (Btu/lb.$^oF$) |
| Water speci c heat capacity, constant volume ($C_{Vw}$) | 0.932 (Btu/lb.$^oF$) |
| Rock speci c heat capacity, constant volume ($C_{Vr}$) | 0.22 (Btu/lb.$^oF$) |

Table 2: Sensitivity Analysis on Nx (Nt=5000 , dt=0.01)

| Nx | CPU Time (sec) | Stability Condition |
|---|---|---|
| 5 | 108 | Stable, very high numerical dispersion |
| 10 | 156 | Stable, high numerical dispersion |
| 30 | 364 | Stable, moderate numerical dispersion |
| 60 | 712 | Some oscillation, low numerical dispersion |
| 150 | 2244 | Unstable, very low numerical dispersion |
| 300 | 5408 | Unstable |

Table 3: Sensitivity Analysis on dt (Nx=30 , Final time=50 Day)

| Nx | CPU Time (sec) | Stability Condition |
|---|---|---|
| 0.01 | 364 | Stable |
| 0.05 | 76 | Stable |
| 0.1 | 36 | No oscillation, over-saturated |
| 0.5 | 8 | No oscillation, over-saturated |
| 1 | 8 | Some oscillation |
| 2 | 8 | More oscillation |
| 5 | { | Nearly singular matrix - no result |

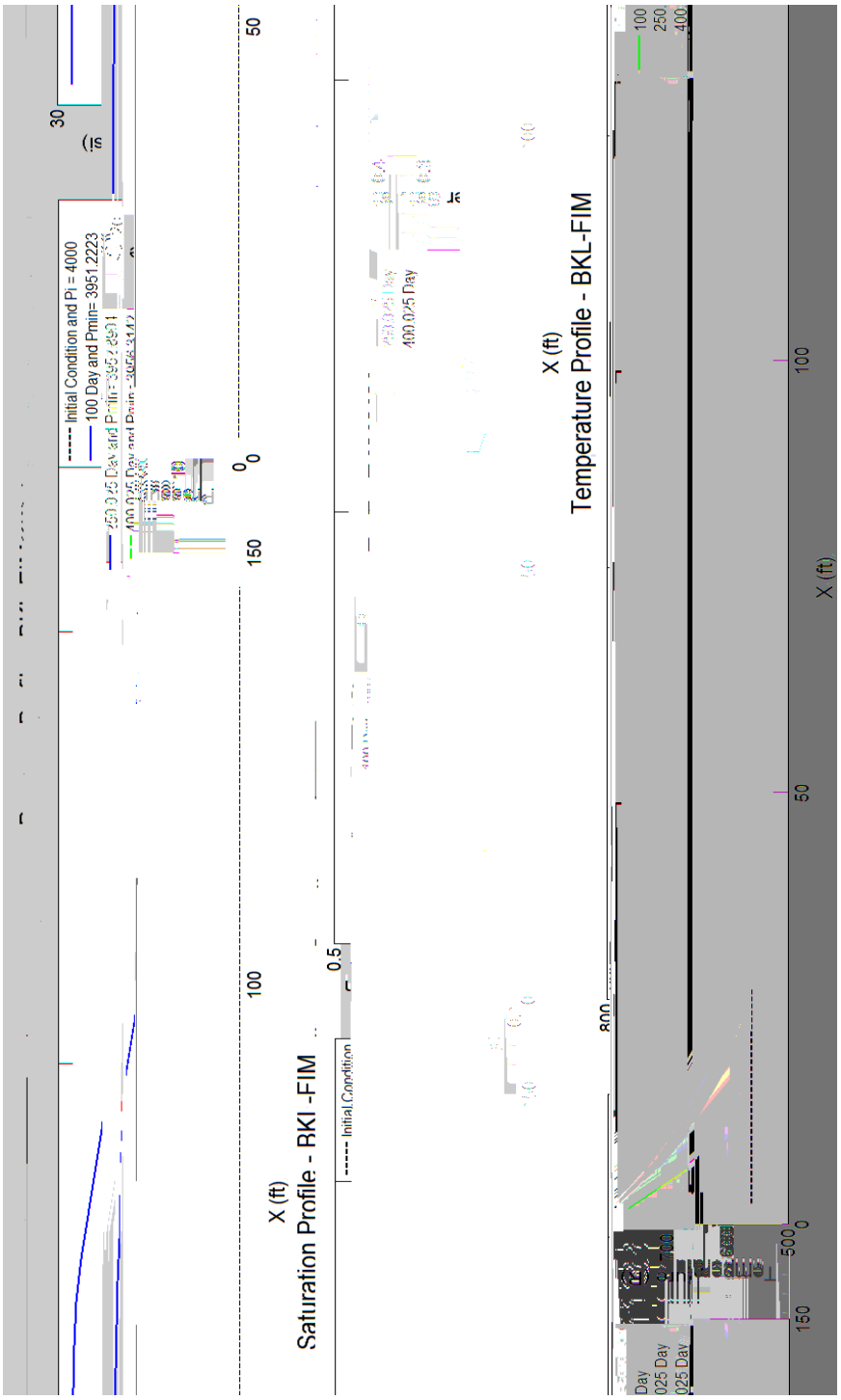



Figure 5.1: The First Model Pressure, Saturation and Temperature Results

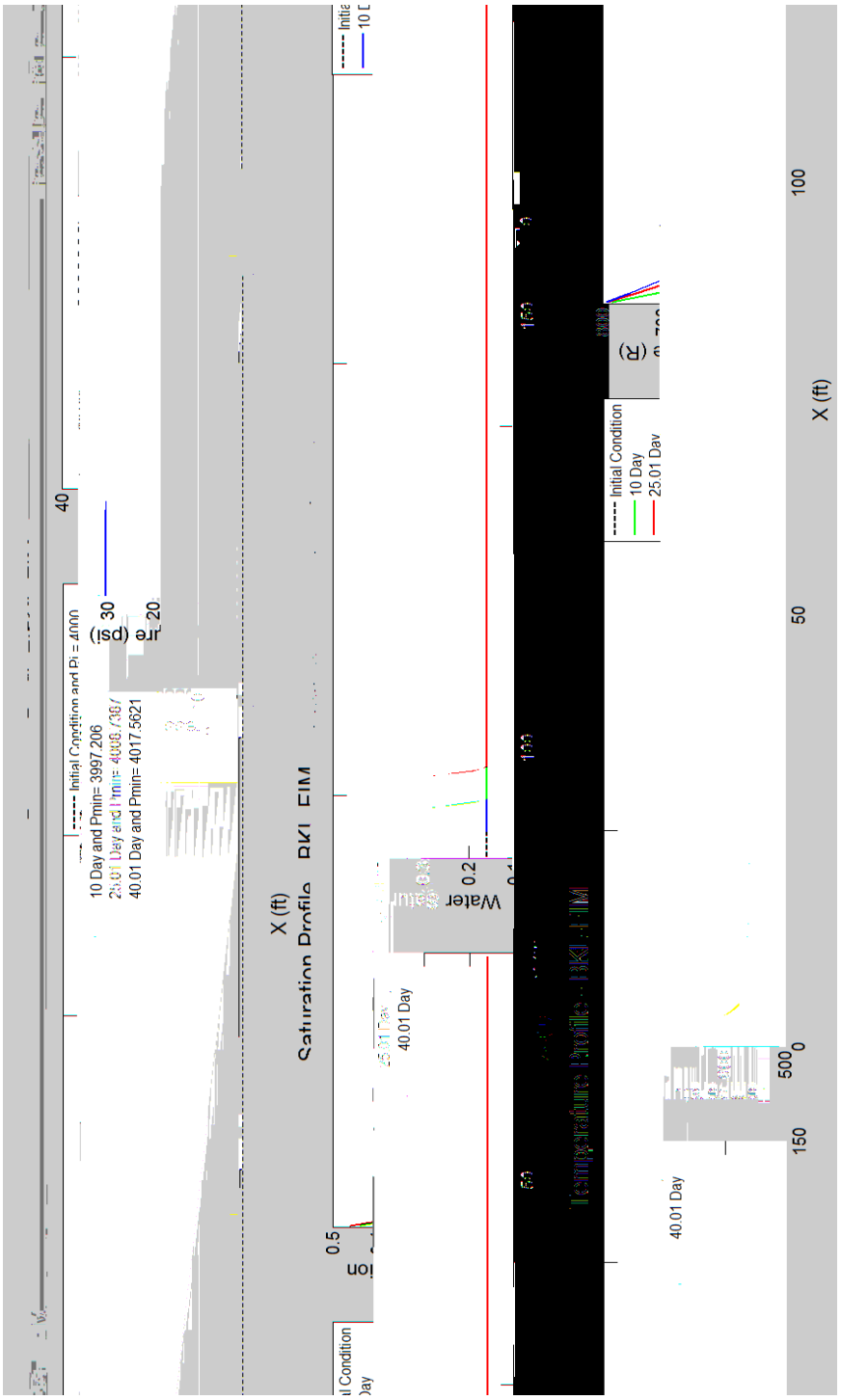
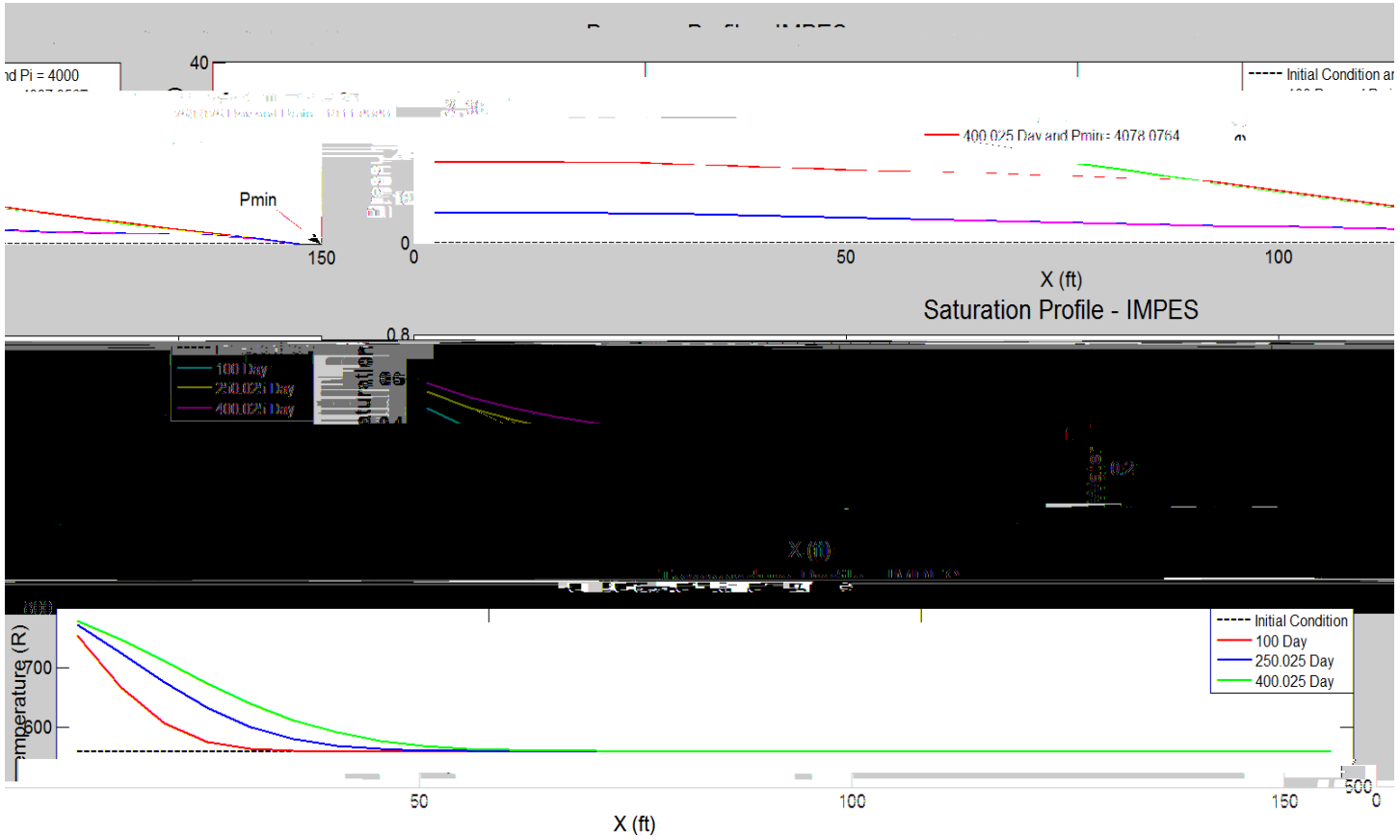



Figure 5.2: The First Model Pressure, Saturation and Temperature Results , Nx=300


Initial Condition ar
400.025 Day and Pmin = 4078.0764
Pmin
150
0
0
50
100
X (ft)
Saturation Profile - IMPES
0.8
100 Day
250.025 Day
400.025 Day
X (ft)
Initial Condition
100 Day
250.025 Day
400.025 Day
700
600
500
0
50
100
150
X (ft)

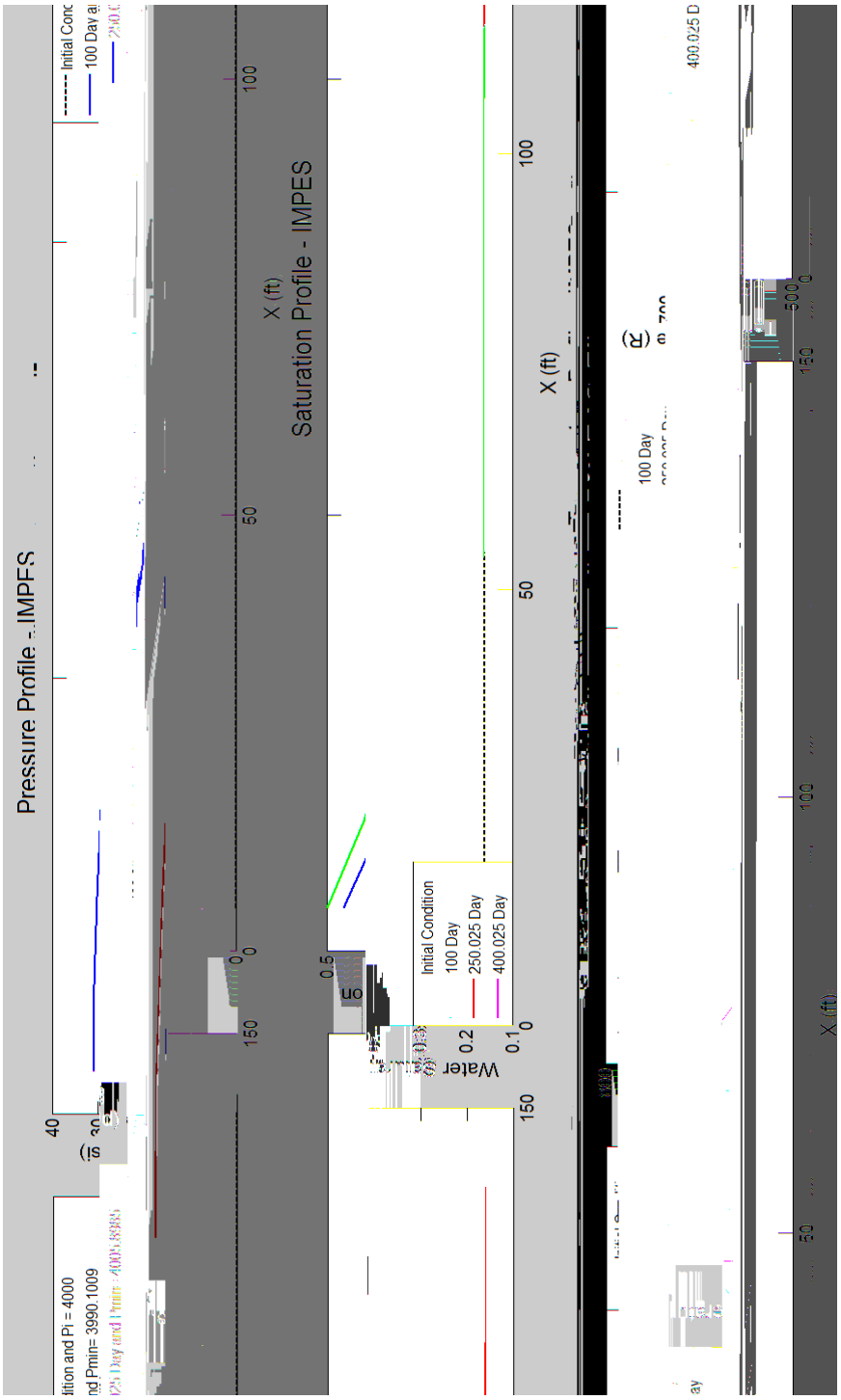
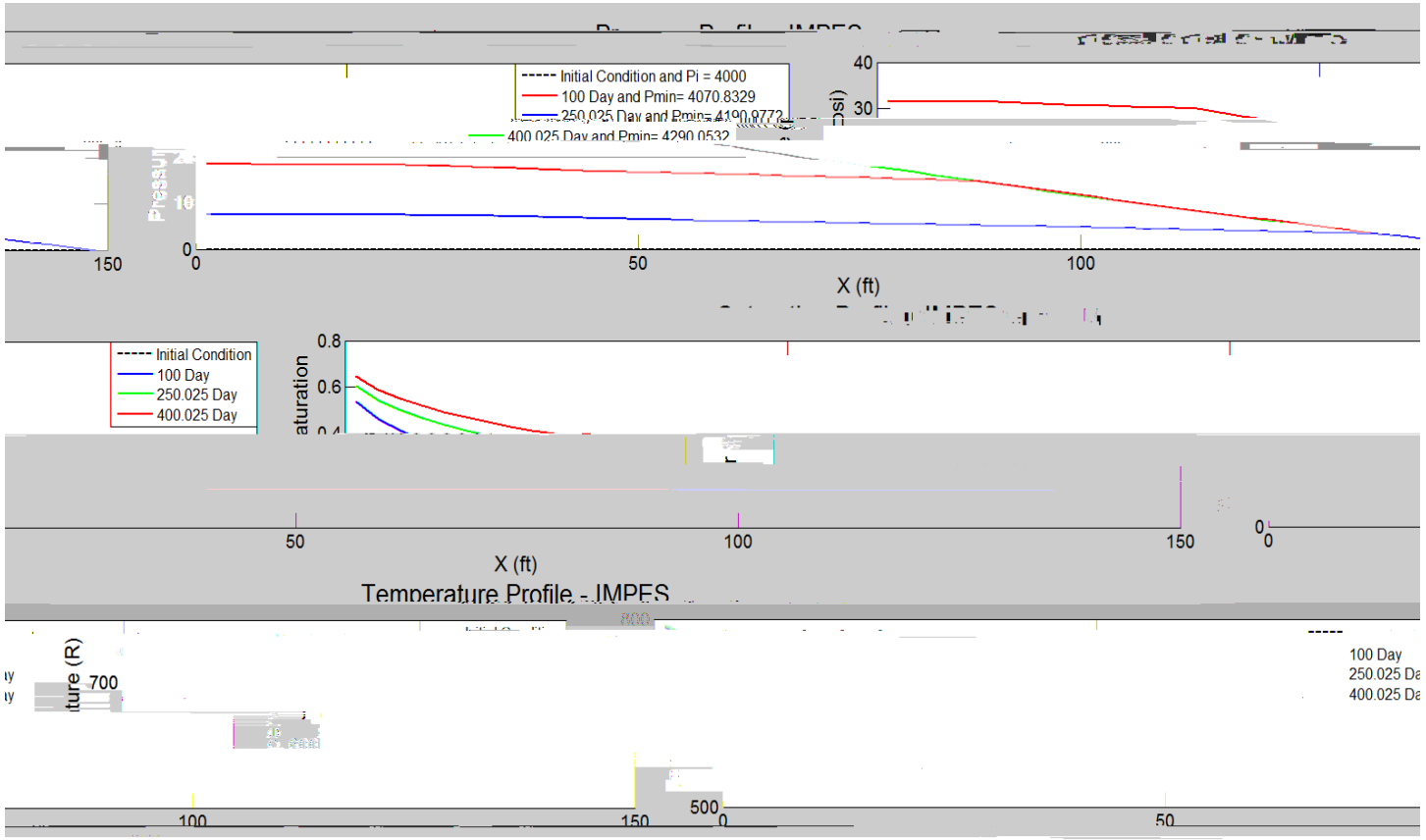



Figure 5.4: Pressure, Saturation and Temperature Pro les for the second model for Nx=15


Initial Condition and Pi = 4000
100 Day and Pmin= 4070.8329
250.025 Day and Pmin= 4190.9772
400.025 Day and Pmin= 4290.0532
40
30
0
150
0
50
100
X (ft)
Initial Condition
100 Day
250.025 Day
400.025 Day
0.8
0.6
50
100
150
0
0
X (ft)
Temperature Profile - IMPES
700
100
150
500
0
50
100 Day
250.025 Da
400.025 Da

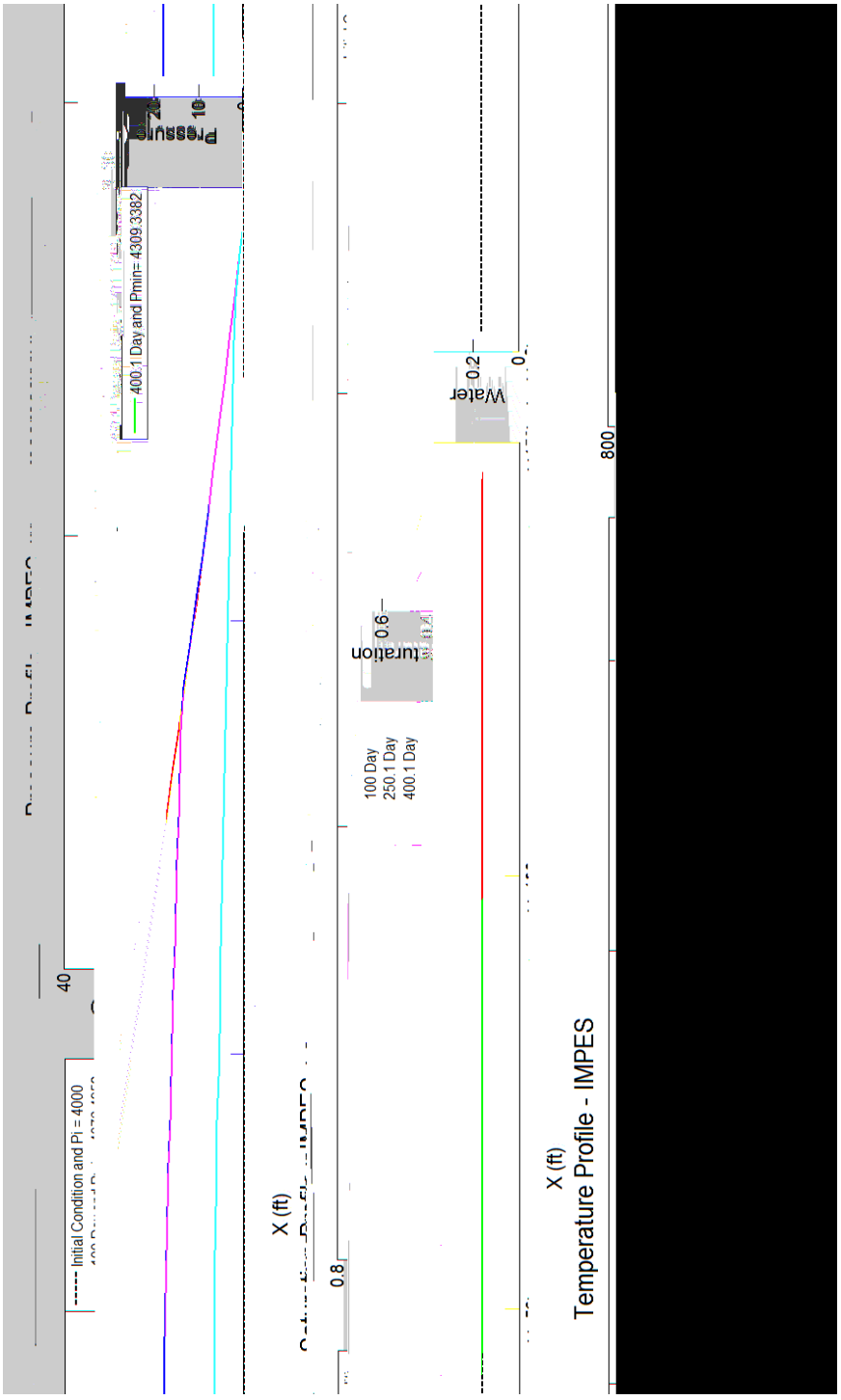



Figure 5.6: The Second Model Pressure, Saturation and Temperature Pro les (dt=0.1 Day)

# 6 Conclusion

In this dissertation two numerical models have been applied to a method of oil recovery, hot water injection. This physical process can be described using oil and water mass conservation equations, energy balance and Buckley-Leverett equations. Hot water injection is modeled by using these equations in order to find pressure, saturation and temperature profiles.

In first model Lax-Wendroff and fully implicit scheme have been used to solve Buckley-Leverett, oil mass and energy balance equations respectively. The model may be less reliable since one of the basic assumptions of Buckley-Leverett equation is considering fluids as incompressible which is not a reasonable assumption in this study. It is worth noticing that

results with the result of this study. For instance, oil and water mass balance equations and energy balance equation can be modeled using fully implicit method for all of them and solve them simultaneously which is a well-known method but, it is really expensive technique in calculation.

Using di erent boundary conditions (well models) to investigate their e ect on the saturation, pressure and temperature distributions in the system.

In rst model, other numerical methods like Warming-Beam, Fromm, etc., can be used to nd the saturation in Buckley-Leverett equation.

# Nomenclature

| | |
|---|---|
| P | Pressure (psia) |
| V | |

# Subscripts and Superscripts

| | |
|---|---|
| | Phase index (o = oil; w = water) |
| w | Water phase |
| o | Oil phase |
| b | Boundary |
| c | Capillary pressure |
| abs | Absolute |
| p | Pore or Pressure |
| r | Relative or Rock |
| t | Total |
| prod | Production |
| inj | Injection |
| gen | Generation |
| cons | Consumption |
| V | Volume |
| or | Residual oil |
| wc | Connate water |
| av | Average |
| ref | Reference |

## References

[1] Tarek Ahmed, Reservoir Engineering Handbook, Second Edition, Gulf Professional Publishing, 2001, pp.184-228, 281-286.

[2] R. E. Ewing, The Mathematics of Reservoir Simulation, SIAM, Philadelphia, (1983)

[3] A. T.Corey,The Interrelation Between Gas and Oil Relative Permeabilities, Producers Monthly, 1954, pp.38-41

[4] www.most.gov.mm/techuni/media/PE