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Correlated observation errors in data assimilation

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Abstract

Data assimilation provides techniques for combining observations and prior model forecasts to create initial conditions for numerical weather prediction (NWP). The relative weighting assigned to each observation in the analysis is determined by its associated error. Remote sensing data usually has correlated errors, but the correlations are typically ignored in NWP. Here we describe three approaches to the treatment of observation error correlations. For an idealised data set, the information content under each simplified assumption is compared to that under the correct correlation specification. Treating the errors as uncorrelated results in a significant loss of information. However, retention of an approximated correlation gives clear benefits.

2 Methods and data

2 Data assimilation

The main aim of variational data assimilation methods is to minimise a cost function which measures the distance of the solution to the background an

2 2 Shannon Information Content

The Shannon Information Content (SIC) is a measure of the reduction of entropy. Entropy physically corresponds to the volume of state space occupied by the probability density function (pdf) describing the knowledge of the state. Assuming all pdfs are Gaussian, [8],

$$
\begin{aligned}
\mathrm{SIC} &= \frac{1}{2}\left|\mathrm{S_A^{-1}B}\right|, &\quad (3)\\
\mathrm{S_A^{-1}} &= \mathrm{H^T R^{-1} H + B^{-1}}, &\quad (4)
\end{aligned}
$$

where $\mathrm{S_A}$ is the analysis error covariance matrix ($S_A(i,j)$ describes the error covariance between components i and j of $\mathrm{x_A}$). The larger the SIC, the greater the reduction in uncertainty in our analysis.

2 2 2 Degrees of freedom of signal

The number of degrees of freedom of signal (dof_S) indicate the number of quantities deemed measured by the observations; the closer dof_S is to the total number of degrees of freedom (dof), the more information the observations have provided.

We have an initial covariance matrix B, and performing an analysis to minimise the variance in observed directions gives us a posterior matrix $\mathrm{S_A}$. The size of the eigenvalues in each matrix represent the size of the uncertainty in the direction of the associated eigenvector; in comparing the eigenvalues of the two, we can determine the reduction in uncertainty.

Take a non-singular square matrix L, as in [4], such that $\mathrm{LBL^T} = \mathrm{I}$ and $\mathrm{LS_AL^T} = \mathrm{S_A}$. This transformation is not unique as we can replace L by $\mathrm{X^TL}$ where X is an orthogonal matrix. Now if we take X to be the matrix of the eigenvectors of $\mathrm{S_A}$, then we simultaneously reduce B to the identity matrix and $\mathrm{S_A}$ to a diagonal matrix of its eigenvalues, ;

$$
\begin{aligned}
\mathrm{X^TLBL^TX} &= \mathrm{X^TX = I},\\
\mathrm{X^TLS_AL^TX} &= \mathrm{X^TS_AX} = \;.
\end{aligned}
$$

4. **Describe R by a truncated eigendecomposition [5];**

$$\mathrm{R} = \mathrm{D}^{1/2}\left(\alpha \mathrm{I} + \sum_{k=1}^{K} (\lambda_k - \alpha)\mathrm{v}_k \mathrm{v}_k^T\right)\mathrm{D}^{1/2} = \mathrm{D}^{1/2}\mathrm{C}\mathrm{D}^{1/2} \qquad (6)$$

$(\lambda_k, \mathrm{v}_k)$ is an eigenvalue, eigenvector pair of C, K is the number of leading eigenpairs used in the approximation, and α is chosen such that trace(R)=trace(D), i.e, so that there is no mis-approximation of the total error variance.

Results

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proach is the approximation of $\mathrm{R_t}$ through its leading eigenpairs; this retains much of the information available even with less than half the available eigenpairs. But, addressing Fisher's concerns [5], we find that spurious long range correlations are present even for larger observation sets.

Although creating a truncated decomposition of $\mathrm{R_t}$ is more costly than the traditional operational approach, it includes some of the correlation structure of $\mathrm{R_t}$ and is still realtively easy to invert. If the computational cost involved in this is not too extensive, then it may be possible to include correlations operationally, leading to a more accurate forecast.

The above results are only currently applicable to the idealised framework in which they have been obtained. Although we have used empirically derived observation errors, the background errors are not realistic. Since the calculations of information are dependent on both B and the idealised observation operator H, a more realistic specification of these would produce more general results. Also, this paper has not addressed the operational feasibility of including correlated observation errors in the data assimilation algorithm. In future work, comparisons will be made using more realistic models, and using different approaches to incorporating correlation structures in the observation error covariance matrix.

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