This paper presents a direct method to determine the uncertainty in pressure, flow and Net Present Value (NPV) of a reservoir, using the time-dependent one phase 2-dimensional reservoir flow equations. The uncertainty in the solution for pressures and ultimately, NPV, is is extended here to the analysis of the risked value of the oil field. The net present value, or NPV, of the field is used to assess risk, and is defined by

$$NPV = \int_0^\infty \|\mathbf{Q}(t)\| e^{-\alpha t} dt, \tag{1}$$

where $\mathbf{Q}(t)$ is the flow of oil at the relevant production well and α is some weighting factor.

In the mathematical modelling of the field in a deterministic case, the flow term $\mathbf{Q}(\mathbf{t})$ may be easily obtained if values for the pressure are known, and or the field flow equations have been solved for at each time-step. For the simple model dealt with in our previous work, the flow can be obtained just by the formula,

$$\mathbf{Q}(t) = -k \cdot \nabla(p), \tag{2}$$

where k is the permeability and p the pressure.

In previous work, [1], we were dealing with cases where uncertainties in the permeabilities caused corresponding uncertainties to propagate though to the numerical solutions for the pressure. We now investigate how these uncertainties propagate to cause uncertainties in the flow and, more importantly, in the NPV.

2 Treatment of Pressure Equations

The earlier study, [1], is restricted to a fairly straightforward two-dimensional model equation (with the implicit assumption that the results obtained may be generalised to the three-dimensional case.) The model equation

$$\gamma \frac{\partial p}{\partial t} - \nabla(k \nabla p) = f(\mathbf{r}, t), \tag{3}$$

is used, where γ is the compressibility, p the pressure, k the permeability, and $f(\mathbf{r}, t)$ is some forcing function.

The uncertainties under consideration are all contained in the permeability, which is assumed to have a mean value function and a permeability autocorrelation function (P.A.F.), defined as

$$\rho(\mathbf{r_1}, \mathbf{r_2}) = \frac{\langle (k(\mathbf{r_1}) - k_0(\mathbf{r_1}))(k(\mathbf{r_2}) - k_0(\mathbf{r_2})) \rangle}{\sigma_k(\mathbf{r_1})\sigma_k(\mathbf{r_2})}, \tag{4}$$

a function of the two spatial positions in the permeability field, $\mathbf{r_1}$ and $\mathbf{r_2}$.

We assume the statistical uncertainties in the permeability can be written as a perturbation about the mean value field,

$$k(\mathbf{r}) = k_0(\mathbf{r}) + k_1(\mathbf{r}),$$

0

By using the previous assumption from [1] that the pressure may be written as a series, we can substitute this, and the perturbation series for the permeability into the equation (9) to give

$$\mathbf{Q} \simeq -\left(k^0 + k^1 + k^2\right) \nabla \left(p_0 + p_1 + p_2\right), \tag{10}$$

where all terms up to and including second order have been retained.

If we now take mean values on either side, we obtain a vector expression for the mean value of the flow,

$$\langle \mathbf{Q} \rangle \simeq -k_0 \nabla p_0 - \langle k_1 \nabla p_1 \rangle - \langle k_2 \rangle \nabla p_0 - k_0 \nabla \langle p_2 \rangle. \tag{11}$$

Also, the covariance of the flow may be written,

$$Cov_q \simeq \langle k_1 k_1 \rangle \left(\nabla p_0\right)^2 + 2k_0 \nabla p_0 \cdot \langle k_1 \nabla p_1 \rangle + (k_0)^2 \langle (\nabla p_1) \cdot (\nabla p_1) \rangle \tag{12}$$

By using the values for pressure already computed with the methods described in [1],we can then proceed to calculate the first two statistical moments for the flow numerically. They only contain statistical information from the pressure terms which is already available from previous considerations. Both these terms can then be used in order to calculate the net present value and its statistical moments up to second order.

It is fairly straightforward, when approaching the problem in a practical way, to approximate equation (9) with a central difference approximation so that it can be written,

$$\mathbf{Q_{ij}} = -k_{ij} \nabla_h p_{ij}. \tag{13}$$

The equation for the mean value of the flow then takes the form

$$\langle \mathbf{Q_{ij}} \rangle \simeq -k_{ij}^0 \nabla_h p_{ij}^0 - \langle k_{ij}^1 \nabla_h p_{ij}^1 \rangle - \langle k_{ij}^2 \rangle \nabla_h p_{ij}^0 - k_{ij}^0 \nabla \langle p_{ij}^2 \rangle, \tag{14}$$

and the equivalent covariance term is,

$$Cov_{q_{ij}} \simeq \langle k_{ij}^1 k_{ij}^1 \rangle \left(\nabla_h p_{ij}^0\right)^2 + 2k_{ij}^0 \nabla_h p_{ij}^0 \cdot \langle k_{ij}^1 \nabla_h p_{ij}^1 \rangle + (k_{ij}^0)^2 \langle (\nabla_h p_{ij}^1) \cdot (\nabla_h p_{ij}^1) \rangle. \tag{15}$$

It is these discretised forms for the statistical moments of the flow that we use when calculating our numerical approximations to the NPV.

4 Net Present Value

To assess the numerical value of the Net Present Value of the systems we are considering, we must first treat it as a time-dependent variable; that is,

$$NPV = \int_0^T \|\mathbf{Q}\| e^{-\alpha t} dt, \tag{16}$$

where . The mean value of this term is then calculated quite straight-

$$T \quad \mathbf{ij} \quad -\alpha t \quad 0$$

$$2 \quad T \quad 0 \quad ij \quad ij \quad 2 \quad -\alpha t$$

$$T \quad -\alpha t \quad 0$$

$$\begin{matrix}0 \\ ij\end{matrix} \quad h \quad \begin{matrix}0 \\ ij\end{matrix}$$

$$-\pi^2 \frac{\langle k \rangle}{\gamma} t$$

We observe the values for the N.P.V. at the centre of the square region. Figure 6.1(a) shows the various mean values for the N.P.V. with different permeability variances, compared with the deterministic solution. The homogeneous mean value of the permeability is 0.2. In Figure 6.1(b) the corresponding relative variances are shown for the N.P.V., for the same permeability variances.

In Figure 6.2, we show the equivalent plots in the case of a smaller permeability mean. Here, $kmean = 0.1$. In Figure 6.3, we show the plots of mean of N.P.V. for a larger mean permeability field with $kmean = 0.4$.

Figure 6.1(a)

Figure 6.1(b)

Figure 6.2

Figure 6.3

In Figure 6.1(a) we can see that the mean values for the N.P.V.s corresponding to the smaller values of the permeability field seem to converge to a similar order of magnitude, but significantly different value, to the deterministic solution ($var(k) = 0.0$). The value for the case where the covariance of the permeability field is large with respect to its mean seems not to show convergence.

This effect is repeated in Figures 6.2 and 6.3, with significant convergence being shown in Figure 6.3 where the mean of the permeability is always larger than the mean value field.

7 onclusions

The types of permeability distribution functions we have considered here have been of limited scope. It is hoped to study more complex instances in future work, such as that with anisotropic correlation lengths and spatially varying mean value fields, as studied in [1].

It is very significant that the mean, or second order approximation to the mean, converges to comparable, but certainly different, values for N.P.V. in cases where the permeability variance is small. This is not true in the cases where significant proportions of the realisations of permeability field lie outside the stability region for the numerical scheme. This, of course is broadly in line with results for the pressure solutions found in [1]

The values for the variance of the N.P.V. do not seem to converge whatever the original choice for permeability variance. This may be due to the fact that the integration is being performed over each instantaneous variance and a true approximation for the variance is not being obtained. More results concerning the variance of the N.P.V will be published in the future.

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